kd education academy (9582701166)

Time : 5 Hour

STD 11 Maths

kd 90+ ch-14 probability

Total Marks : 230

*** Choose the right answer from the given options. [1 Marks Each]** **[120]**

1. A person write 4 letters and addresses 4 envelopes. If the letters are placed in the envelopes at random, then the probability that all letters are not placed in the right envelopes, is

(A) $\frac{1}{4}$ (B) $\frac{11}{24}$ (C) $\frac{15}{24}$ (D) $\frac{23}{24}$

Ans. :

d. $\frac{23}{24}$

Solution:

Total number of ways of placing four letters in 4 envelops = 4 = 24

All the letters can be dispatched in the right envelops in only one way. Therefore, the probability that all the letters are placed in the right envelops is $\frac{1}{24}$.

Hence, probability that all the letters are not placed in the right envelops $= 1 - \frac{1}{24} = \frac{23}{24}$

2. One card is drawn from a pack of 52 cards.The probability of getting a 10 of black suit is:

(A) $\frac{1}{26}$ (B) $\frac{1}{13}$ (C) $\frac{3}{26}$ (D) None

Ans. :

a. $\frac{1}{26}$

Solution:

Favourable number of outcomes, with 10 of black suit = 2

Total number of outcomes = 52

Thus, probabilit

$= \frac{2}{52} = \frac{1}{6}$

3. A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, the probability that it is rusted or is a nail is:

(A) $\frac{3}{16}$ (B) $\frac{5}{16}$ (C) $\frac{11}{16}$ (D) $\frac{14}{16}$

Ans. :

c. $\frac{11}{16}$

Solution:

If the numbers of nails and nuts are 6 and 10, respectively, then the numbers of rusted nails and rusted nuts are 3 and 5, respectively.

Total number of items = 6 + 10 = 16

Total number of rusted items = 3 + 5 = 8

Total number of ways of drawing one item $= {}^{16}C_1$

Let R and N be the events where both the items drawn are rusted items and nails, respectively.

R and N are not mutually exclusive events, because there are 3 rusted nails.

P(either a rusted item or a nail) $= P(R \cup N)$

$= P(R) + P(N) - P(R \cap N)$

$= \frac{{}^{6}C_1}{{}^{16}C_1} + \frac{{}^{8}C_1}{{}^{16}C_1} - \frac{{}^{3}C_1}{{}^{16}C_1}$

$= \frac{6}{16} + \frac{8}{16} - \frac{3}{16} = \frac{11}{16}$

4. Poonam buys a fish from a shop for her aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish. What is the probability that the fish taken out is a male fish?

(A) $\frac{5}{13}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{5}{14}$

Ans. :

a. $\frac{5}{13}$

Soultion:

Favourable outcome (Getting a male fish) = 5

Total number of outcomes (Male and female fish) =13

Probabilit $= \frac{5}{13}$

5. A box contains 3 red, 3 white and 3 green balls. A ball is selected at random. Find the probability that the ball picked up is neither a white nor a red ball:

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Ans. :

b. $\frac{1}{3}$

Solution:

Total number of outcomes = 9

Favourable outcomes (the ball is neither white nor red) = 3

Probability $= \frac{3}{9} = \frac{1}{3}$

6. Three integers are chosen at random from the first 20 integers. The probability that their product is even is:

(A) $\frac{2}{19}$ (B) $\frac{3}{29}$ (C) $\frac{17}{19}$ (D) $\frac{4}{19}$

Ans. :

c. $\frac{17}{19}$

Solution:

Number of ways in which we can choose three distinct integers from 20 integers $^{20}C_3 = 1140$

We know that, if we take three odd numbers, there product will always be an odd number.

Out of 20 consecutive integers, 10 are even and 10 are odd integers.

Number of ways in which we can choose three distinct odd integers from 10 odd integers $= \ ^{10}C_3 = 120$

P(product is even) = 1 - P(product is odd),

$= 1 - \frac{120}{1140} = \frac{1140-120}{1140} = \frac{1020}{1140} = \frac{17}{19}$

7. Choose the correct answer.

Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is:

(A) $\frac{1}{5}$ (B) $\frac{4}{5}$ (C) $\frac{1}{30}$ (D) $\frac{5}{9}$

Ans. :

d. $\frac{5}{9}$

Solution:

We have digite 0, 2, 3, 5.

Number of divisible by 5 if unit place digit is '0' or '5'

If unit place is '0' then first three places can be filled in 3! ways.

If unit place is '5' then first place can be filled in two ways and second and thried place can be filled in 2! ways.

So, number of numbers ending with digit '5' is 2 × 2! = 4

$\therefore$ Total number of numbers divisible by 5 = 3! + 4 = 110 = n(E)

Also total number of numbers = 3 × 3! = 18

$\therefore$ Required probability $= \frac{10}{18} = \frac{5}{9}$

8. 20 cards are numbered from 1 to 20. If one card is drawn at random, what is the probability that the number on the card is a prime number?

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) 5

Ans. :

b. $\frac{2}{5}$

Solution:

Let E be the event of getting a prime number.
E = {2, 3, 5, 7, 11, 13, 17, 19} Hence, P(E)
$\frac{8}{20} = \frac{2}{5}$.

9. Four persons are selected at random out of 3 men, 2 women and 4 children. The probability that there are exactly 2 children in the selection is:

(A) $\frac{11}{21}$ (B) $\frac{9}{21}$ (C) $\frac{10}{21}$ (D) None of these

Ans. :

c. $\frac{10}{21}$

Solution:

There are nine persons (three men, two women and four children) out of which four persons can be selected in ${}^{9}C_{4} = 126$ ways.

Total number of elementary events = 126

Exactly two children means selecting two children and two other people from three men and two women.

This can be done in ${}^{4}C_{2} \times {}^{5}C_{2}$ways.

Favourable number of elementary events = ${}^{4}C_{2} \times {}^{5}C_{2} = 60$

So, required probability $= \frac{60}{120} = \frac{10}{21}$

10. All the three face cards of spades are removed from a well-shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting black face card:

(A) $\frac{6}{49}$ (B) $\frac{3}{49}$ (C) $\frac{5}{49}$ (D) $\frac{4}{49}$

Ans. :

b. $\frac{3}{49}$

Solution:

Total number of possibilities Total number of possibilities = 49 (Since, 3 cards of spade are removed)

Number of black face cards = 3 (3 cards of clubs)

Thus, required probability

$= \frac{3}{49}$

11. The probabilities of happening of two events A and B are 0.25 and 0.50 respectively. If the probability of happening of A and B together is 0.14, then probability that neither A nor B happens is:

(A) 0.39 (B) 0.29 (C) 0.11 (D) None of these.

Ans. :

a. 0.39

Solution:

$P(A) = 0.25, P(B)=0.50$ and $P(A \cap B) = 0.14$

$\therefore$ Required Probability $= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B]$

$= 1 - [0.25 + 0.50 - 0.14]$

$= 1 - 0.61 = 0.39$

12. Choose the correct answer.

While shuffling a pack of 52 playing cards, 2 are accidentally dropped. Find the probability that the missing cards to be of different colours:

(A) $\frac{29}{52}$ (B) $\frac{1}{2}$ (C) $\frac{26}{51}$ (D) $\frac{27}{51}$

Ans. :

c. $\frac{26}{51}$

Solution:

We know that out of 52 playing cards 26 are of red and 26 are of black colour.

$\therefore$ P(both cards of differents colour)

$= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$

$= 2 \times \frac{26}{52} \times \frac{26}{51} = \frac{6}{51}$

13. A and B are two events such that P (A) = 0.25 and P (B) = 0.50. The probability of both happening together is 0.14. The probability of both A and B not happening is

(A) 0.39 (B) 0.2 (C) 0.11 (D) none of these.

Ans. :

a. 0.39

Solution:

$p(A) = 0.25$ and $P(B)=0.50$

$P(A \cap B) = 0.14$

$\therefore$ Required probability $= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [0.25 + 0.50 - 0.14]$

$= 1 - 0.61 = 0.39$

14. What is the probability of selecting a vowel in the word "PROBABILITY"?

(A) $\frac{2}{11}$ (B) $\frac{3}{11}$ (C) $\frac{4}{11}$ (D) $\frac{5}{11}$

Ans. :

b. $\frac{3}{11}$

15. A card is drawn at random from a pack of 100 cards numbered 1 to 100. The probability of drawing a number which is a square is:

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{1}{10}$ (D) None of these

Ans. :

c. $\frac{1}{10}$

Solution:

Clearly, the sample space is given by

S = {1, 2, 3, 4, 5 ....97, 98, 99, 100}

$\therefore$ n(S) = 100

Let E = event of getting a square.

Then E = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}

$\therefore$ n(E) = 10

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{10}{100} = \frac{1}{10}$

16. If events A and B are independent and P(A) = 0.15, P(A ∪ B) = 0.45, then P(B)=:

(A) 136 (B) 176 (C) 196 (D) 236

Ans. :

b. 176

Solution:

Given, P(A) = 0.15, P(A ∪ B) = 0.45

We have P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

and P(A ∩ B) = P(A). P(B)

Therefore, 0.45 = 0.15 + P(B) - 0.15 P(B)

⇒ 0.30 = 0.85 P(B)

⇒ P(B) = 8530 = 176

17. Two dice are thrown simultaneously. Find the probability of getting a multiple of 2 on first dice and a multiple of 3 on the second dice.

(A) $\frac{4}{6}$ (B) $\frac{2}{6}$ (C) $\frac{1}{6}$ (D) $\frac{1}{36}$

Ans. :

c. $\frac{1}{6}$

Solution:

Total cases = 6 × 6

Let A be the event of getting a multiple of 2 on first dice and a multiple of 3 on the second dice.

Hence, A = {(2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6)} n(A) = 66

$\therefore P(A) = \frac{6}{36} = \frac{1}{6}$

18. 6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is

(A) $\frac{1}{432}$ (B) $\frac{12}{431}$ (C) $\frac{1}{132}$ (D) none of these

Ans. :

Total number of ways in which 6 boys and 6 girls can sit in a row = 12

Consider 6 girls as one group, then 6 boys and one group can arrange in 7 ways.

Now, 6 girls in the group can arrange among themselves in 6.

So, the number of ways in which all the girls sit together is 7 × 6

∴ P(all girls sit together)

$$= \frac{\text{Number of ways in which all girls sit together}}{\text{Total Number of ways in which 6 boys and 6 girls sit in a row}}$$

$$= \frac{7 \times 6}{12} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{12 \times 11 \times 10 \times 9 \times 8} = \frac{1}{132}$$

Hence, the correct answer is option (c).

19. The probability of getting a total of 10 in a single throw of two dices is:

(A) $\frac{1}{9}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{5}{36}$

Ans. :

b. $\frac{1}{12}$

Solution:

When two dices are thrown, there are (6 × 6) = 36 outcomes.

The set of all these outcomes is the sample space, given by

S = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

i.e. n(S) = 36

Let E be the event of getting a total score of 10.

Then E = {(4, 6), (5, 5), (6, 4)}

∴ n(E) = 3

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

20. Five persons entered the lift cabin on the ground floor of an 8 floor house. Suppose that each of them independently and with equal probability can leave the cabin at any floor beginning with the first, then the probability of all 5 persons leaving at different floor is:

(A) $\frac{{}^7P_5}{7^5}$ (B) $\frac{7^5}{{}^7P_5}$ (C) $\frac{6}{{}^6P_5}$ (D) $\frac{{}^5P_5}{5^5}$

Ans. :

a. $\dfrac{{}^7P_5}{7^5}$

Solution:

Since, it is an eight - storey building.

So, there are 7 possible options for them in 7 floors in total if ground floor is not considered.

Hence, total possible outcomes = $7\times 7\times 7\times 7\times 7 = 7^5$

Thus, number of ways in which 5 persons can leave from seven floors differently $= {}^7P_5$

Required probability $= \dfrac{{}^7P_5}{7^5}$

21. If three dice are throw simultaneously, then the probability of getting a score of 5 is:

(A) $\frac{5}{216}$ (B) $\frac{1}{6}$ (C) $\frac{1}{36}$ (D) None of these

Ans. :

c. $\dfrac{1}{36}$

Solution:

When three dice are thrown together, the sample space S associated with the random experiment is given by,

S = {(1, 1, 1), (1, 1, 2), (1, 1, 3) ...(6, 6, 5), (6, 6, 6)}

Clearly, total number of elementary events n(S) = 216

Let A be the event of getting a total score of 5.

Then A = { (1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1)}

$\therefore$ Favourable number of elementary events = 6

i.e. n(A) = 6

Hence, required probability $= \frac{6}{216} = \frac{1}{36}$

22. The probability that a leap year will have 53 Fridays or 53 Saturdays is:

(A) $\frac{2}{7}$ (B) $\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{1}{7}$

Ans. :

b. $\dfrac{3}{7}$

Solution:

We know that a leap year has 366 days (i.e. 7 × 52 + 2) = 52 weeks and 2 extra days .

The sample space for these 2 extra days is given below:

S = {(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)} There are 7 cases.

$\therefore n(S) = 7$

Let E be the event that the leap year has 53 Fridays or 53 Saturdays.

E = { (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)}

i.e. n(E) = 3

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{7}$

Hence, the probability that a leap year has 53 Fridays or 53 Saturdays is $\frac{3}{7}$.

23. There are 30 tickets numbered from 1 to 30 in a box . A ticket is drawn at random. What is the probability that the ticket drawn bears an odd number?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{4}$

Ans. :

a. $\frac{1}{2}$

Solution:

Total number of outcomes = 30

Favourable outcomes (odd number on the ticket) = 15

Probability $= \frac{15}{30} = \frac{1}{2}$

24. Find the sample space for choosing a prime number less than 2020 at random.

(A) 2, 3, 5, 7, 11, 13, 17, 19 (B) 2, 3, 4, 5, 7, 11, 13, 17, 19

(C) 2, 3, 5, 7, 11, 13, 17, 19, 20 (D) 2, 3, 5, 7, 11, 13, 17, 19, 15

Ans. :

a. 2, 3, 5, 7, 11, 13, 17, 19

Solution:

Sample space is the collection of all possible events.

So, sample space for choosing a prime number less than 20 = 2, 3, 5, 7, 11, 13, 17, 19.

25. Two numbers are chosen from {1, 2, 3, 4, 5, 6} one after another without replacement. Find the probability that the smaller of the two is less than 4.

(A) $\frac{4}{5}$ (B) $\frac{1}{15}$ (C) $\frac{1}{5}$ (D) $\frac{14}{15}$

Ans. :

a. $\frac{4}{5}$

Solution:

Total number of ways of choosing two numbers out of six

$= {}^{6}C_{6} = \frac{(6 \times 2)}{2} = 3 \times 5 = 15$

If smaller number is chosen as 3 then greater has choice are 4, 5, 6 So, total choices = 3

If smaller number is chosen as 2 then greater has choice are 3, 4, 5, 6 So, total choices = 4

If smaller number is chosen as 1 then greater has choice are 2, 3, 4, 5, 6 So, total choices = 5

Total favourable case = 3 + 4 + 5 = 12

Now, required probability

$= \frac{12}{15} = \frac{4}{5}$

26. A die is thrown then find the probability of getting a number greater than 3.

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) 0

Ans. :

b. $\frac{1}{2}$

Solution:

Sample space = 1, 2, 3, 4, 5, 6

a no >3 in sample space = 4, 5, 6 = 3

probability of getting no greater than $3 = \frac{1}{2}$

27. Let A and B are two mutually exclusive events and if P(A) = 0.5 and $P(\bar{B})$ =0.6 then P(AUB) is:

(A) 0 (B) 1 (C) 0.6 (D) 0.9

Ans. :

d. 0.9

Solution:

Given, A and B are two mutually exclusive events.

So, $P(A \cap B) = 0$

Again given P(A) = 0.5 and $P(\bar{B}) = 0.6$

$P(B) = 1 - P(\bar{B}) = 1 - 0.6 = 0.4$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B)$

$\Rightarrow P(A \cup B) = 0.5 + 0.4 = 0.9$

28. If two coins are tossed then find the probability of the events that at the most one tail turns up:

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Ans. :

d. $\frac{3}{4}$

Solution:

The sample space of 2 coins tossed = (h, h), (h, t), (t, h), (t, t)

for having atmost one tail we need = (h, t), (t, h), (h, h)

Thus the probability is $\frac{3}{4}$

29. Choose the correct answer.

6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is:

(A) $\frac{1}{432}$ (B) $\frac{12}{431}$ (C) $\frac{1}{132}$ (D) none of these.

Ans. :

c. $\frac{1}{132}$

Solution:

If all the girls sit together, then we consider it as 1 group

G	G	G	G	G	G	

1group ⟷ 6 boys

$\therefore$ Total number of arrangement of 6 + 1 = 7 persons in a row = 7! And the girls also interchanged their places with 6! Ways.

$\therefore$ Required probability $= \frac{6!7!}{12!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 7!}{12 \times 11 \times 10 \times 9 \times 8 \times 7!} = \frac{1}{132}$

30. One of the two events must occur. If the chance of one is $\frac{2}{3}$ of the other, then odds in favour of the other are

(A) 1 : 3 (B) 3 : 1 (C) 2 : 3 (D) 3 : 2

Ans. :

d. 3 : 2

Solution:

Let $P(B) = X$

Than, $P(A) = \frac{2x}{3}$

$P(A) + P(B) = x + \frac{2x}{3} = \frac{5x}{3}$

$\Rightarrow \frac{5x}{3} = 1$ ($\therefore$ They are exhaustive events)

$\Rightarrow x = \frac{3}{5}$

Now, $P(A) = \frac{2}{5}$ and $P(B) = \frac{3}{5}$

$\therefore$ odd in favour of B $= \frac{\frac{3}{5}}{\frac{1-3}{5}} = \frac{3}{2} = 3 : 1$

31. Two dice are thrown together. The probability that neither they show equal digits nor the sum of their digits is 9 will be:

(A) $\frac{13}{15}$ (B) $\frac{13}{18}$ (C) $\frac{1}{9}$ (D) $\frac{8}{9}$

Ans. :

b. $\frac{13}{18}$

Solution:

When two dices are thrown, there are (6 × 6) = 36 outcomes.

The set of all these outcomes is the sample space is given by

S = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

$\therefore n(S) = 36$

Let E be the event of getting the digits which are neither equal nor give a total of 9.

Then E' = event of getting either a doublet or a total of 9

Thus, E' = {{1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (3, 6), (4, 5), (5, 4), (6, 3)}

i.e. n(E') = 10

$P(E^{'}) = \frac{n(E^{'})}{n(E)} = \frac{10}{36} = \frac{5}{18}$

Hence, required probability P(E) = 1 - P(E')

$= 1 - \frac{5}{18} = \frac{13}{18}$

32. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. The probability that none of the balls drawn is blue is:

(A) $\frac{10}{21}$ (B) $\frac{11}{21}$ (C) $\frac{2}{7}$ (D) $\frac{5}{7}$

Ans. :

a. $\frac{10}{21}$

Solution:

Total number of balls = 2 + 3 + 2 = 7

Two balls are drawn.

Now, P(none of them is blue) $= \frac{^5C_2}{^7C_2}$

$= \frac{\left\{\frac{(5\times4)}{(2\times1)}\right\}}{\left\{\frac{(7\times6)}{(2\times1)}\right\}}$

$= \frac{(5\times4)}{(7\times6)}$

$= \frac{(5\times2)}{(7\times3)}$

$= \frac{10}{21}$

33. Two unbiased coins are tossed simultaneously. Find the probability of getting at least one head.

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) None of these

Ans. :

c. $\frac{3}{4}$

Solution:

If two unbiased coins are tossed simultaneously, then the sample space will be S.

S: {H H, H T, T H, T T} n(S) = 4n

E: At least one head is obtained {H H, H T, T H} n(E) = 3n(E) = 3

Hence, P(At least one head) $= \frac{3}{4}$

34. If $\frac{(1-3P)}{2}, \frac{(1+4P)}{3}, \frac{(1+P)}{6}$ are the probabilities of three mutually exclusive and exhaustive events, then the set of all values of *p* is:

(A) $(0, 1)$ (B) $\left(\frac{-1}{4}, \frac{1}{3}\right)$ (C) $\left(0, \frac{1}{3}\right)$ (D) $(0, \infty)$

Ans. :

b. $\left(\frac{-1}{4}, \frac{1}{3}\right)$

Solution:

$P(A) = \frac{(1-3P)}{2}$

$P(B) = \frac{(1+4P)}{3}$

$P() = \frac{(1+P)}{6}$

The events are mutually exclusive and exhaustive.

$\therefore P(A \cup B \ \cup C) = P(A) + P(B) + P(C) = 1$

$\Rightarrow 0 \le P(A) \le 1, 0 \le P(B) \le 1, 0 \le P(C) \le 1$

$\Rightarrow 0 \le \frac{1-3P}{2} \le 1, 0 \le \frac{1-4P}{3} \le 1, 0 \le \frac{1-P}{6} \le 1$

$\Rightarrow \frac{-1}{3} \le P \le \frac{1}{3} \ldots (1)$

$\frac{-1}{4} \le P \le \frac{1}{2} \ldots (2)$

and $-1 \le P \le 5 \ldots (3)$

The common solution of (1), (2), and (3) is $\frac{-1}{4} \le P \le \frac{1}{3}$

$\therefore$ The set values of P are $\left(\frac{-1}{4}, \frac{1}{3}\right)$

35. A pack of cards contains 4 aces, 4 kings, 4 queens and 4 jacks. Two cards are drawn at random. The probability that at least one of them is an ace is

(A) $\frac{1}{5}$ (B) $\frac{3}{16}$ (C) $\frac{9}{20}$ (D) $\frac{1}{9}$

Ans. :

c. $\dfrac{9}{20}$

Solution:

We have:

$\text{P(both are aces)} = \dfrac{{}^{4}C_{2}}{{}^{16}C_{2}}$

$= \dfrac{4}{16} \times \dfrac{3}{15} = \dfrac{1}{20}$

$\text{P(one are ace)} = \dfrac{{}^{4}C_{1} \times {}^{12}C_{1}}{{}^{16}C_{2}} = \dfrac{2}{5}$

$\therefore \text{P(at least one are ace)} = \dfrac{1}{20} + \dfrac{2}{5} = \dfrac{9}{20}$

36. Two dice are thrown simultaneously. The probability of getting a pair of aces is

(A) $\frac{1}{36}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) none of these

Ans. :

a. $\dfrac{1}{36}$

Solution:

When two dice are thrown simultaneously, the sample space associated with the random experiment is given by:

S = {(1, 1), (1, 2), (1, 3), (6, 4), (6, 5), (6, 6)}

Clearly, total number of elementary events = 36

Let A be the event of getting a pair of aces.

Then A = {(1, 1)}

$\therefore n(A) = 1$

Hence, required probability $= \dfrac{n(A)}{n(S)} = \dfrac{1}{36}$

37. Two dice are thrown simultaneously. The probability of obtaining total score of seven is:

(A) $\frac{5}{36}$ (B) $\frac{6}{36}$ (C) $\frac{7}{36}$ (D) $\frac{8}{36}$

Ans. :

b. $\dfrac{6}{36}$

Solution:

When two dices are thrown, there are (6 × 6) = 36 outcomes.

The set of all these outcomes is the sample space given by

S = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)
$\therefore$ n(S) = 36
Let E be the event of getting a total score of 7.
Then E = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}
$\therefore$ n(E) = 6
Hence, required probability $= \frac{n(E)}{n(S)} = \frac{6}{36}$

38. One card is drawn from a pack of 52 cards. The probability that it is the card of a king or spade is:

(A) $\frac{1}{26}$ (B) $\frac{3}{26}$ (C) $\frac{4}{13}$ (D) $\frac{3}{13}$

Ans. :

c. $\frac{4}{13}$

Solution:

If A and B denote the events of drawing a king and a spade card, respectively, then event A consists of four sample points, whereas event B consists of 13 sample points.

Thus, $P(A) = \frac{4}{52}$ and $P(B) = \frac{13}{52}$

The compound event (A n B) consists of only one sample point, king of spade.

So, $P(A \cap B) = \frac{1}{52}$

By addition theorem , we have:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

Hence, the probability that the card drawn is either a king or a spade is given by $\frac{4}{13}$.

39. Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that this number has the same digits?

(A) $\frac{1}{16}$ (B) $\frac{16}{25}$ (C) $\frac{1}{645}$ (D) $\frac{1}{25}$

Ans. :

The given digits are 0, 2, 4, 6, 8.

___	___	___
Hundreds	Tens	Ones

Now, there are 4 ways to fill the hundreds place (0 cannot occupy the hundreds place), 5 ways to fill the tens place and 5 ways to fill the ones place.

Total number of 3 digit numbers formed using the given digits = 4 × 5 × 5 = 100

The three digit numbers formed using given digits that have the same digits are 222, 444, 666 and 888

Number of 3 digit numbers that have the same digits = 4

$\therefore$ P(three digit number formed has the same digits)

$$\frac{\text{Number of 3 digits numbers that have the same digits}}{\text{Total number of 3 digit numbers formed using the given digits}}$$

$= \frac{4}{100} = \frac{1}{25}$

Hence, the correct answer is option (d).

40. Sample space is a set of ..... of an experiment.

(A) All possible outcomes (B) Selected outcomes

(C) Both (D) None of these

Ans. :

a. All possible outcomes

Solution:

A sample space is usually denoted using set notation, and the possible outcomes are listed as elements in the set. For example, if the experiment is tossing a coin, the sample space is typically the set {head, tail}, i.e all possible outcomes.

41. Three numbers are chosen from 1 to 20. The probability that they are not consecutive is:

(A) $\frac{186}{190}$ (B) $\frac{187}{190}$ (C) $\frac{188}{190}$ (D) $\frac{18}{^{20}C_3}$

Ans. :

Number of ways to choose three numbers from 1 to 20 $= {}^{20}C_3 = 1140$

Now, the set of three consecutive numbers from 1 to 20 are (1, 2, 3), (2, 3, 4), (3, 4, 5), ..., (18, 19, 20).

So, the number of ways to choose three numbers from 1 to 20 such that they are consecutive is 18.

P(three numbers choosen are consecutive)

$$= \frac{\text{Number of ways to choose three consecutive numbers from 1 to 20}}{\text{Number of ways to choose three numbers from 1 to 20}}$$

$= \frac{18}{^{20}C_3} = \frac{18}{1140} = \frac{3}{190}$

$\therefore$ P(three numbers choosen are not consecutive) = 1 - P(three numbers choosen are consecutive) $= 1 - \frac{3}{190} = \frac{187}{190}$

Hence, the correct answer is option (b).

42. If A, B, C are three mutually exclusive and exhaustive events of an experiment such that $3\,P(A) = 2P(B) = C$, then P(A) is equal to:

(A) $\frac{1}{11}$ (B) $\frac{2}{11}$ (C) $\frac{5}{11}$ (D) $\frac{6}{11}$

Ans. :

Let 3 P(A) = 2 P(B) = P(C) = p.

Then, $P(A) = \frac{P}{3}, P(B) = \frac{P}{2}$ and $P(C) = P$

It is given that A, B, C are three mutually exclusive and exhaustive events.

$\therefore P(A) + P(B) + P(C) = 1$

$= 0$ and $[P(A \cup B) = P(B \cap C) = P(C \cap A) = P(A \cup B \cup C = 1]$

$\Rightarrow \frac{P}{3} + \frac{p}{2} + P = 1$

$\Rightarrow \frac{11P}{6} = 1$

$\Rightarrow P = \frac{6}{11}$

$\therefore P(A) = \frac{P}{3} = \frac{\frac{6}{11}}{3} = \frac{2}{11}$

Hence, the correct answer is option (b).

43. Two dice are thrown together. The probability that at least one will show its digit greater than 3 is:

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{8}$

Ans. :

b. $\frac{3}{4}$

Solution:

When two dice are thrown, there are (6 × 6) = 36 outcomes. The set of all these outcomes is the sample space, given by

S = (1, 1) , (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1) , (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1) , (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1) , (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1) , (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1) , (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

i.e. n(S) = 36

Let E be the event of getting at least one digit greater than 3.

Then E = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 1) , (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),

(5, 1) , (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1) , (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }

$\therefore n(E) = 27$

Hence, required probability $= \frac{27}{36} = \frac{3}{4}$

44. One coin is tossed once. Find the probability of getting A tail.

(A) $\frac{1}{2}$ (B) 1 (C) Data insufficient (D) None of these

Ans. :

a. $\frac{1}{2}$

Solution:

There are 2 possible outcomes of a throw of coin.

$P(\text{tail}) = \frac{1}{2}$

45. Choose the correct answer.

The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is:

(A) 0.4 (B) 0.8 (C) 1.2 (D) 1.6

Ans. :

c. 1.2

Solution:

We have, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$

$\therefore\ P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$

$\Rightarrow 0.6 = P(A) + P(B) - 0.2$

$\Rightarrow P(A) + P(B) = 0.8$

$\therefore\ P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B)$

$= 2 - [P(A) + P(B)] = 2 - 0.8 = 1.2$

46. If a coin is tossed till the first head appears, then what will be the sample space?

(A) {H} (B) {TH}

(C) {T, TH, HHT, HHHT, ...} (D) {H, TH, TTH, TTTH, ...}

Ans. :

d. {H, TH, TTH, TTTH, ...}

Solution:

S: {H, TH, TTH, TTTH, ...} infinte elements.

If for the first toss only, we would have got the head, we have stop there itself.

47. One card is drawn from a pack of 52 cards. The probability of getting a jack card is:

(A) $\frac{1}{13}$ (B) $\frac{2}{13}$ (C) $\frac{3}{13}$ (D) $\frac{4}{13}$

Ans. :

a. $\frac{1}{13}$

Solution:

Favourable number of outcomes i.e., numbers of jack cards = 4

Total number of outcomes = 52

Thus, probability $= \frac{4}{52} = \frac{1}{13}$

48. Two unbiased coins are tossed simultaneously. Find the probability of getting at most one head.

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{1}{3}$

Ans. :

c. $\frac{3}{4}$

Solution:

Since, Total possibilities are = {H H, H T, T H, T T}

no. of cases with atmost one head are = {H T, T H, T T}

$= \frac{3}{4}$

49. Probability is 0.45 that a dealer will sell at least 20 television sets during a day, and the probability is 0.74 that he will sell less that 24 televisions. The probability that he will sell 20, 21, 22 or 23 televisions during the day, is:

(A) 0.19 (B) 0.32 (C) 0.21 (D) None of these

Ans. :

a. 0.19

Solution:

Let A be the event that the sale is at least 20 televisions, i.e. 20, 21, 22,... and B be the event that sale is less than 24 i.e. 0.1.2.3...23.

Then A∩B will denote the sale of 20, 21, 22 and 23 televisions, We are given P(A) = 0.45 and P(B) = 0.74.

It is required to find P(A ∩ B).

Also P(A ∪ B) = P(sale of 0, 1, 2, 3,...,20, 21, 22, 23 televisions) = P(S) = 1.

From addition rule, required probability is P(A ∩ B) = P(A) + P(B) - P(A ∪ B) = 0.45 + 0.74 - 1 = 0.19.

50. Six boys and six girls sit in a row at random. The probability that the boys and girls sit alternatively is:

(A) $\frac{1}{462}$ (B) $\frac{11}{462}$ (C) $\frac{5}{51}$ (D) $\frac{7}{123}$

Ans. :

a. $\frac{1}{462}$

Solution:

Given, 6 boys and 6 girls sit in a row at random.

Then, the total number of arrangement of 6 boys and 6 girls = arrangement of 12 persons = 12! Now, boys and girls sit alternatively.

So, the total number of arrangement = 2 × 6! × 6!

Now, P(boys and girls sit alternatively) $= \frac{(2 \times 6! \times 6!)}{12!}$

$$= \frac{(2 \times 6! \times 6!)}{(12! \times 11!)}$$
$$= \frac{(5! \times 6!)}{11!}$$
$$= \frac{(5 \times 4 \times 3 \times 2 \times 1 \times 6!)}{(11 \times 10 \times 9 \times 8 \times 7 \times 6!)}$$
$$= \frac{(5 \times 4 \times 3 \times 2)}{(11 \times 10 \times 9 \times 8 \times 7)}$$
$$= \frac{(4 \times 3)}{(11 \times 9 \times 8 \times 7)}$$
$$= \frac{3}{(11 \times 9 \times 2 \times 7)}$$
$$= \frac{1}{(11 \times 9 \times 2 \times 7)}$$
$$= \frac{1}{462}$$

51. Choose the correct answer.
If the probabilities for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is:

(A) > 0.5 (B) 0.5 (C) ≤ 0.5 (D) 0

Ans. :

c. ≤ 0.5

Solution:

Given, P(A fail) = 0.2

and P(B fail) = 0.3

$\therefore$ P(either A or B fail) $\leq$ P(A fail) + P(B fail)

$\leq 0.2 + 0.3$

≤ 0.5

52. Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is:

(A) $\frac{1}{5}$ (B) $\frac{4}{5}$ (C) $\frac{1}{30}$ (D) $\frac{5}{9}$

Ans. :

The given digits are 0, 2, 3 and 5.

_____	_____	_____	_____
Thousands	Hundreds	Tens	Ones

Now, there are 3 ways to fill the thousands place (0 cannot occupy the thousands place), 3 ways to fill the hundreds place, 2 ways to fill the tens place and 1 way to fill the ones place.

Total number of four digit numbers formed = 3 × 3 × 2 × 1 = 18

We know that a number is divisible by 5 if it ends in 0 or 5.

When 0 is at the ones place,

Number of four digits numbers divisible by 5 formed = 3 × 2 × 1 = 6

When 5 is at the ones place,

Number of four digits numbers divisible by 5 formed = 2 × 2 × 1 = 4 (0 cannot occupy the thousands place)

Total number of four digit numbers divisible by 5 = 6 + 4 = 10

$\therefore$ P(four digit number formed is divisible by 5)

$= \frac{\text{Total Number of four digit numbers divisible by 5}}{\text{Total Number of w4 digit numbers formed}}$

$= \frac{10}{18} = \frac{5}{9}$

Hence, the correct answer is option (d).

53. Two unbiased coins are tossed simultaneously. The probability of getting at least one head is:

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) none

Ans. :

c. $\frac{3}{4}$

Solution:

Favourable number of outcomes, getting at least one head = 3[H H, H T, T H]

Total number of outcomes = 4[H H, H T, T H, T T]

Probability $= \frac{3}{4}$

54. A die is rolled, then the probability that an even number is obtained is:

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Ans. :

a. $\frac{1}{2}$

Solution:

When a die is rolled, total number of outcomes = 6 (1, 2, 3, 4, 5, 6)

Total even number = 3 (2, 4, 6)

So, the probability that an even number is obtained

$= \frac{3}{6} = \frac{1}{2}$

55. A box contains 10 good articles and 6 with defects. One item is drawn at random. The probability that it is either good or has a defect is:

(A) $\frac{64}{64}$ (B) $\frac{49}{64}$ (C) $\frac{40}{64}$ (D) $\frac{24}{64}$

Ans. :

a. $\frac{64}{64}$

Solution:

Let A be the event of drawing one good article whereas B be the event of drawing one defected article.

Here,

$P(A) = \frac{10}{10+6} = \frac{10}{16}$

$P(B) = \frac{6}{10+6} = \frac{6}{16}$

The events A and B are mutually exclusive. Thus, the required probability,

$P(A \cup B) = P(A) + P(B)$

$\Rightarrow P(A \cup B) = \frac{10}{16} + \frac{6}{16} = \frac{16}{16} = 1$

Hence, the correct option is (a).

56. The probability that the leap year will have 53 sundays and 53 monday is:

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{7}$ (D) $\frac{1}{7}$

Ans. :

d. $\frac{1}{7}$

Solution:

In a leap year, total number of days = 366 days.

In 366 days, there are 52 weeks and 2 days.

Now two days may be

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday

Now in total 7 possibilities, Sunday and Monday both come together is 1 time.

So probabilities of 53 Sunday and Monday in a leap year

$= \frac{1}{17}$

57. Choose the correct answer.

Three numbers are chosen from 1 to 20. Find the probability that they are not consecutive:

(A) $\frac{186}{190}$ (B) $\frac{187}{190}$ (C) $\frac{188}{190}$ (D) $\frac{18}{{}^{20}C_3}$

Ans. :

b. $\frac{187}{190}$

Solution:

Since, the set of three consecutive numbers from 1 to 20 are (1, 2, 3), (2, 3, 4), (3, 4, 5), ... , (18, 19, 20), i.e., 18

P(numbers are consecutive)

$= \frac{18}{{}^{20}C_3} = \frac{18}{\frac{20 \times 19 \times 18}{3!}} = \frac{3}{190}$

P(three number are not consecutive)

$= 1 - \frac{3}{190} = \frac{187}{190}$

58. In tossing a coin, the chance of throwing head and tail alternatively in 3 successive trials is:

(A) $\frac{1}{4}$ (B) $\frac{1}{6}$ (C) $\frac{1}{5}$ (D) $\frac{1}{48}$

Ans. :

a. $\frac{1}{4}$

Solution:

Favourable outcomes ={H T H, T H T} = 2 outcomes

Total number of outcomes = 8

Probability $= \frac{2}{8} = \frac{1}{4}$

59. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn. Then the probability that they both are diamonds is:

(A) $\frac{84}{452}$ (B) $\frac{48}{452}$ (C) $\frac{84}{452}$ (D) $\frac{84}{452}$

Ans. :

d. $\frac{84}{452}$

Solution:

Total number of cards = 52 and one card is lost. Case 1: if lost card is a diamond card

Total number of cards = 51 Number of diamond cards = 12 Now two cards are drawn.

P(both cards are diamonds) $\frac{={}^{12}C_2}{{}^{51}c_2}$

Total number of cards = 52 and one card is lost.

Case 2: If lost card is not a diamond card Total number of cards = 51

Number of diamond cards = 13Now two cards are drawn.

P(both cards are diamonds) $\frac{={}^{13}C_2}{{}^{51}c_2}$

Now probability that both cards are diamond $\frac{={}^{12}C_2}{{}^{51}c_2} + \frac{={}^{13}C_2}{{}^{51}c_2}$

$= \frac{{}^{12}C_2 + {}^{13}C_2}{{}^{51}C_2}$

$\left\{\frac{(12\times11)}{(2\times1)} + \frac{(13\times12)}{(2\times1)}\right\}\left\{\frac{(51\times50)}{(2\times1}\right\}$

$= \frac{(12\times11\times+13\times12)}{(51\times50)}$

$= \frac{288}{2550}$

$= \frac{96}{850}$ (288 and 2550 divided by 3)

$= \frac{48}{425}$ (96 and 850 divided by 2)

So probability that both cards are diamond is $\frac{48}{425}$

60. If the integers m and n are chosen at random between 1 and 100, then the probability that the number of the from $7^m + 7^n$ is divisible by 5 equals:

(A) $\frac{1}{4}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{1}{49}$

Ans. :

a. $\frac{1}{4}$

Solution:

Since m and n are selected between 1 and 100,

Hence total sample space = 100 × 100

Again, 71 = 7, 72 = 49, 73 = 343, 74 = 2401, 75 = 16807, etc

Hence 1, 3, 7 and 9 will be the last digit in the power of 7.

Now, favourable number of case are

→ 1,1 1,2 1,3 1,100

2,1 2,2 2,3 2,100

3,1 3,2 3,3 3,100

100,1 100,2 100,3 100,100

Now, for m = 1, n = 3, 7, 11, ..., 97

So, favourable cases = 25

Again for m = 2, n = 4, 8, 12, ..., 100

So, favourable cases = 25

Hence for every m, favourable cases = 25

So, total favourable cases = 100 × 25 Required Probability

$= \frac{(100 \times 25)}{(100 \times 100)}$

$= \frac{25}{100}$

$= \frac{1}{4}$

61. If S is the sample space and $P(A) = \frac{1}{3}P(B)$ and $S = A \cup B$ where A and B are two mutually exclusive events, then P (A) =

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{3}{8}$

Ans. :

a. $\frac{1}{4}$

Solution:

Let $P(B) = P$

Than $P(A) = \frac{1}{3}P$

Since A and B are two mutually exclusive events, we have:

$A \cup B = S$

$\Rightarrow P(A \cup B) = P(S)$

$\Rightarrow P(A \cup B) = 1$

$\Rightarrow P(A) + (B) = 1$

$\Rightarrow \frac{1}{2}P + P = 1$

$\Rightarrow \frac{4p}{3} = 1$

$\therefore P(A) = \frac{1}{3}P = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$

62. Two dice are thrown the events A, B, C are as follows A: Getting an odd number on the first die. B: Getting a total of 7 on the two dice. C: Getting a total of greater than or equal to 8 on the two dice. Then AUB is equal to

(A) 15 (B) 17 (C) 19 (D) 21

Ans. :

d. 21

Solution:

When two dice are thrown, then total outcome = 6 × 6 = 36

A: Getting an odd number on the first die.

A = {(1, 1), (1, 2), (1, 3), (1, 4),(1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4),(3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4),(5, 5), (5, 6)}

Total outcome = 18 B: Getting a total of 7 on the two dice.

B = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)} Total outcome = 6

C: Getting a total of greater than or equal to 8 on the two dice.

C = {(2, 6), (3, 5), (3, 6), (4, 4),(4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6),(6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} Total outcome = 15

Now $n(A \cup B) = n(A) + n(B) - n(A \cup B)$

$\Rightarrow n(A \cup B) = 18 + 6 - 3$

$\Rightarrow n(A \cup B) = 21$

63. A die is rolled. What is the probability that an even number is obtained?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Ans. :

a. $\frac{1}{2}$

Solution:

When a die is rolled, total number of outcomes = 6 (1, 2, 3, 4, 5, 6)

Total even number = 3 (2, 4, 6)

So, the probability that an even number is obtained

$= \frac{3}{6} = \frac{1}{2}$

64. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting a face card.

(A) $\frac{1}{13}$ (B) $\frac{1}{26}$ (C) $\frac{3}{13}$ (D) None of these

Ans. :

c. $\frac{3}{13}$

Solution:

Total number of outcomes = 52

Favourable outcomes (A face card) = 12

Probability $= \frac{12}{52} = \frac{3}{13}$

65. In a simultaneous throw of two dice what is the probability of getting a doublet ?

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$

Ans. :

a. $\frac{1}{6}$

Solution:

Total number of possibilities = 36

Number of doublet = 6

Thus, probability

$= \frac{6}{36} = \frac{1}{6}$

66. The probabilities of three mutually exclusive events A, B and C are given by $\frac{2}{3}, \frac{1}{4}$ and $\frac{1}{6}$ respectively. The statement

(A) Is true. (B) Is false.

(C) Nothing can be said. (D) Could be either.

Ans. :

b. Is false.

Solution:

Since the events A, B and C are mutually exclusive, we have:

$P(A \cup B \cup C) = \frac{2}{3} + \frac{1}{4} + \frac{1}{6} = \frac{13}{12} > 1$

which is not possible.

Hence, the given statement is false.

67. If the probability for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is:

(A) > 0.5 (B) 0.5 (C) ≤ 0.5 (D) 0

Ans. :

Let X and Y be two events given by,

X : A fails in an examination

Y : B fails in an examination

P(A fails) = P(X) = 0.2

P(B fails) = P(Y) = 0.3

Now, P(either A or B fails) $= \mathrm{P}(\mathrm{X} \cup \mathrm{Y})$

We know that,

$= \mathrm{P}(\mathrm{X} \cup \mathrm{Y}) \leq \mathrm{P}(\mathrm{X}) + \mathrm{P}()\mathrm{Y} = 0.2 + 0.3 = 0.5$

$\Rightarrow \mathrm{P}(\mathrm{X} \cup \mathrm{Y}) \leq 0.5$

$\therefore \mathrm{P}(\text{either A or B fails}) \leq 0.5$

Hence, the correct answer is option (c).

68. Two dice are thrown:

P is the event that the sum of the scores on the uppermost faces is a multiple of 6.

Q is the event that the sum of the scores on the uppermost faces is at least 10.

R is the event that same scores on both dice.

Which of the following pairs is mutually exclusive?

(A) P, Q (B) P, R (C) Q, R (D) None of these

Ans. :

d. None of these

Solution:

Possibilities ofP, (3, 3),(6, 6),(1, 5),(5, 1),(4, 2),(2, 4)

Possibilities of Q:(5, 5),(5, 6),(6, 5),(6, 6)

Possibilities of R:(1, 1),(2, 2),(3, 3),(4, 4),(5, 5),(6, 6)

Thus, the possibilities are neither exhaustive, nor mutually exclusive nor these are complementary probabilities.

69. All possible outcomes of a random experiment forms the:

(A) Events (B) Sample space (C) Both (D) None of these

Ans. :

b. Sample space

Solution:

Sample Space is the set of all possible outcomes of an experiment. It is denoted by S.Examples:

When a coin is tossed, S = {H, T} where H = Head and T = Tail

When a dice is thrown, S = {1, 2 , 3, 4, 5, 6}

When two coins are tossed, S = {HH, HT, TH, TT} where H = Head and T = Tail

70. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals:

(A) $\frac{1}{2}$ (B) $\frac{7}{15}$ (C) $\frac{2}{15}$ (D) $\frac{1}{3}$

Ans. :

b. $\frac{7}{15}$

Solution:

While placing 7 while balls in a row, total gaps = 8

3 black balls can be placed in 8 gaps

$= C = \frac{(8\times7\times6)}{(3\times2\times1)} = 8 \times 7 = 56$

So, the total number of ways of arranging white and black balls such that no two black balls are adjacent = 56 × 3! × 7!

Actual number of arrangement possible with 7 white and 3 black balls = (7 + 3)! = 10!

So, the required Probability

$= \frac{(56\times3!\times7!)}{10!}$

$= \frac{(56\times3!\times7!)}{(10\times9\times8\times7!)}$

$= \frac{(56\times3!)}{(10\times9\times8)}$

$= \frac{(56\times3\times2\times1)}{(10\times9\times8)}$

$= \frac{(7\times3\times2\times1)}{(10\times9)}$

$= \frac{(7\times2)}{(10\times3)}$

$= \frac{7}{(5\times3)}$

$= \frac{7}{15}$

71. If 4-digit numbers greater than 5000 are randomly formed from the digits 0, 1, 3, 5 and 7, then the probability of forming a number divisible by 5 when the digits are repeated is:

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$

Ans. :

b. $\frac{2}{5}$

Solution:

Given digits are 0, 1, 3, 5, 7

Now we have to form 4 digit numbers greater than 5000.

So leftmost digit is either 5 or 7.

When digits are repeated

Number of ways for filling left most digit = 2

Now remaining 3 digits can be filled = 5 × 5 × 5

So total number of ways of 4 digits greater than 5000 = 2 × 5 × 5 × 5 = 250

Again a number is divisible by 5 if the unit digit is either 0 or 5. So there are 2 ways to fill the unit place.

So total number of ways of 4 digits greater than 5000 and divisible by 5 = 2 × 5 × 5 × 2 = 100

Now probability of 4 digit numbers greater than 5000 and divisible by 5

$= \frac{100}{250}$

$= \frac{2}{5}$

72. Choose the correct answer.

In a non-leap year, the probability of having 53 tuesdays or 53 wednesdays is:

(A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) none os these.

Ans. :

a. $\frac{1}{7}$

Solution:

There are 365 days in non-leap year and there are 7 days in a week

$\therefore\ 365 \div 7 = 52$ weeks + 1 days

So, this day may be Tuesday or Wednesday.

So, the required probability $= \frac{1}{7}$

73. What is the total number of sample spaces when a die is thrown 2 times?

(A) 6 (B) 12 (C) 18 (D) 36

Ans. :

d. 36

Solution:

The possible outcomes when a die is thrown are 1, 2, 3, 4, 5, and 6.

Given, a die is thrown two times.

Then, the total number of sample space = (6 × 6)

= 36

74. Let S be the sample space of all five digit numbers.If p is the probability that a randomly selected number from S, is a multiple of 7 but not divisible by 5 , then $9\,p$ is equal to.

(A) 1.0146 (B) 1.2085 (C) 1.0285 (D) 1.1521

Ans. : c

Sol. $n(S) =$ all 5 digit nos $= 9 \times 10^4$

A: no is multiple of 7 but not divisible by 5

Smallest 5 digit divisible by 7 is 10003

Largest 5 digit divisible by 7 is 99995

$\therefore 99995 = 10003 + (n-1)7 \quad n = 12857$

Numbers divisible by 35

$99995 = 10010 + (P-1)35 \Rightarrow P = 2572$

$\therefore$ Numbers divisible by 7 but not by 35 are

$12857 - 2572 = 10285$

$\therefore P = \frac{10285}{90000}$

$\therefore 9\,P = 1.0285$

75. The probability, that in a randomly selected $3-$digit number at least two digits are odd, is

(A) $\frac{19}{36}$ (B) $\frac{15}{36}$ (C) $\frac{13}{36}$ (D) $\frac{23}{36}$

Ans. : a

$=$ exactly two digits are odd $+$ exactly there 3 digits are odd

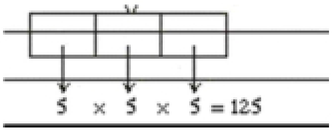
For exactly three digits are odd

For exactly two digits odd :

If 0 is used then : $2 \times 5 \times 5 = 50$

If 0 is not used then : ${}^{3}C_{1} \times 4 \times 5 \times 5 = 300$

Required Probability $= \frac{475}{900} = \frac{19}{36}$



76. If the probability that a randomly chosen 6-digit number formed by using digits 1 and 8 only is a multiple of 21 is p, then 96 p is equal to

(A) 30 (B) 33 (C) 40 (D) 43

Ans. : b

$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Divisible by 21 when divided by 3 .

Case - I : All 1 → (1)

Case - II : All 8 → (1)

Case - III : 3 ones and 3 eights

$\frac{6!}{3! \times 3!} = 20$

Required probability $\therefore p = \frac{22}{64}$

$96p = 96 \times \frac{22}{64} = 33$

77. In an examination, there are 10 true-false type questions. Out of 10 , a student can guess the answer of 4 questions correctly with probability $\frac{3}{4}$ and the remaining 6 questions correctly with probability $\frac{1}{4}$. If the probability that the

student guesses the answers of exactly 8 questions correctly out of 10 is $\frac{27k}{4^{10}}$, then k is equal to

(A) 598 (B) 487 (C) 412 (D) 479

Ans. : d

$A = \{1, 2, 3, 4\} : P(A) = \frac{3}{4} \rightarrow$ Correct

$B = \{5, 6, 7, 8, 9, 10\}; P(B) = \frac{1}{4}$ Correct

8 Correct

$(4, 4): {}^4C_4\left(\frac{3}{4}\right)^4 \cdot {}^6C_4 \cdot \left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right)^2$

$(3, 5): {}^4C_3\left(\frac{3}{4}\right)^3 \cdot \left(\frac{1}{4}\right)^1 \cdot {}^6C_5\left(\frac{1}{4}\right)^5 \cdot \left(\frac{3}{4}\right)$

$(2, 6): {}^4C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2 \cdot {}^6C_6\left(\frac{1}{4}\right)^6$

$\text{Total} = \frac{1}{4^{10}}\left[3^4 \times 15 \times 3^2 + 4 \times 3^3 \times 6 \times 3 + 6 \times 3^2\right]$

$= \frac{27}{4^{10}}[2.7 \times 15 + 72 + 2]$

$\Rightarrow K = 479$

78. Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1) = \frac{2+3p}{6}, P(E_2) = \frac{2-p}{8}$ and $P(E_3) = \frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(p_1 + p_2)$ is equal to.

(A) $\frac{2}{3}$ (B) $\frac{5}{3}$ (C) $\frac{5}{4}$ (D) 1

Ans. : d

$0 \le P\left(E_i\right) \le 1$ for $i = 1, 2, 3$

$-2/3 \le p \le 1$

E_1, E_2, E_3 are mutually exclusive

$P\left(E_1\right) + P\left(E_2\right) + P\left(E_3\right) \le 1$

$2/3 \le p \le 1$

$p_1 = 1, p_2 = 2/3$

$p_1 + p_2 = 5/3$

79. The probabilities of a student getting I, II and III division in an examination are respectively $\frac{1}{10}, \frac{3}{5}$ and $\frac{1}{4}$. The probability that the student fails in the examination is

(A) $\frac{197}{200}$ (B) $\frac{27}{100}$ (C) $\frac{83}{100}$ (D) None of these

Ans. : d

(d) A denote the event getting I;

B denote the event getting II;

C denote the event getting III;

and D denote the event getting fail.

Obviously, these four event are mutually exclusive and exhaustive,

therefore $P(A)+P(B)+P(C)+P(D)=1$

$\Rightarrow P(D)=1-0.95=0.05$.

80. The chance of India winning toss is $3/4$. If it wins the toss, then its chance of victory is $4/5$ otherwise it is only $1/2$. Then chance of India's victory is

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{3}{40}$ (D) $\frac{29}{40}$

Ans. : d

(d) There are two mutually exclusive cases for the event.

$A=$ India wins the toss and wins the match.

$B=$ India losses the toss and wins the match

$\therefore$ Required probability

$=P(A)+P(B)=\frac{3}{4}\times\frac{4}{5}+\frac{1}{4}\times\frac{1}{2}=\frac{29}{40}$.

81. A six faced dice is so biased that it is twice as likely to show an even number as an odd number when thrown. It is thrown twice. The probability that the sum of two numbers thrown is even, is

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

Ans. : d

(d) **Trick :** As we know, the sum will be either even or odd but even is more likely to occur than odd (given).

Therefore, the probability is greater than $\frac{1}{2}$ which is given in only one option.

82. Cards are drawn one by one without replacement from a pack of 52 cards. The probability that 10 cards will precede the first ace is

(A) $\frac{241}{1456}$ (B) $\frac{164}{4165}$ (C) $\frac{451}{884}$ (D) None of these

Ans. : b

(b) There are four aces and 48 other cards.

Therefore the required probability

$=\frac{48\cdot 47\cdot\ldots\ldots\cdot 39}{52\cdot 51\cdot\ldots\ldots\cdot 43}\cdot\frac{4}{42}=\frac{164}{4165}$.

83. A box contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept aside the first. This process is repeated till all the

balls are drawn from the box. The probability that the balls drawn are in the sequence of 2 black, 4 white and 3 red is

(A) $\frac{1}{1260}$ (B) $\frac{1}{7560}$ (C) $\frac{1}{126}$ (D) None of these

Ans. : a

(a) The required probability

$= \frac{2}{9} \times \frac{1}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} \times 1 \times 1 \times 1 = \frac{1}{1260}.$

84. A problem of mathematics is given to three students whose chances of solving the problem are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. The probability that the question will be solved is

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{4}{5}$ (D) $\frac{3}{5}$

Ans. : d

(d) The probability of students not solving the problem are $1 - \frac{1}{3} = \frac{2}{3},\ 1 - \frac{1}{4} = \frac{3}{4}$ and $1 - \frac{1}{5} = \frac{4}{5}$

Therefore the probability that the problem is not solved by any one of them $= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$

Hence the probability that problem is solved $= 1 - \frac{2}{5} = \frac{3}{5}$.

85. The probability of hitting a target by three marksmen are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that one and only one of them will hit the target when they fire simultaneously, is

(A) $\frac{11}{24}$ (B) $\frac{1}{12}$ (C) $\frac{1}{8}$ (D) None of these

Ans. : a

(a) Here $P(A) = \frac{1}{2},\ P(B) = \frac{1}{3},\ P(C) = \frac{1}{4}$

Hence required probability

$= P(A)P(\bar{B})P(\bar{C}) + P(\bar{A})P(B)P(\bar{C}) + P(\bar{A})P(\bar{B})P(C).$

86. The probability of A, B, C solving a problem are $\frac{1}{3}, \frac{2}{7}, \frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, the probability that exactly one of them will solve it, is

(A) $\frac{25}{168}$ (B) $\frac{25}{56}$ (C) $\frac{20}{168}$ (D) $\frac{30}{168}$

Ans. : b

(b) Here $p_1 = \frac{1}{3},\ p_2 = \frac{2}{7}$ and $p_3 = \frac{3}{8}$

$\Rightarrow q_1 = \frac{2}{3},\ q_2 = \frac{5}{7}$ and $q_3 = \frac{5}{8}$

Required probability $= p_1 q_2 q_3 - q_1 p_2 q_3 + q_1 q_2 p_3$.

87. A man and his wife appear for an interview for two posts. The probability of the husband's selection is $\frac{1}{7}$ and that of the wife's selection is $\frac{1}{5}$. What is the probability that only one of them will be selected

(A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) None of these

Ans. : b

(b) The probability of husband is not selected $= 1 - \frac{1}{7} = \frac{6}{7}$

The probability that wife is not selected $= 1 - \frac{1}{5} = \frac{4}{5}$

The probability that only husband selected $= \frac{1}{7} \times \frac{4}{5} = \frac{4}{35}$

The probability that only wife selected $= \frac{1}{5} \times \frac{6}{7} = \frac{6}{35}$

Hence required probability $= \frac{6}{35} + \frac{4}{35} = \frac{10}{35} = \frac{2}{7}$.

88. There are 4 envelopes with addresses and 4 concerning letters. The probability that letter does not go into concerning proper envelope, is

(A) $\frac{19}{24}$ (B) $\frac{21}{23}$ (C) $\frac{23}{24}$ (D) $\frac{1}{24}$

Ans. : c

(c) Required probability is $1 - P$ (they go in concerned envelopes)

$= 1 - \frac{1}{4\,!} = \frac{23}{24}$.

89. A number is chosen from first 100 natural numbers. The probability that the number is even or divisible by 5, is

(A) $\frac{3}{4}$ (B) $\frac{2}{3}$ (C) $\frac{4}{5}$ (D) $\frac{3}{5}$

Ans. : d

(d) $2, 4, 6, 8, 10 \ldots\ldots.$ i.e., fifty even and ten divisible by 5 like $5, 15, 25 \ldots\ldots.$

as $(10, 20, 30 \ldots\ldots.)$ have been considered.

Hence required probability $= \frac{50+10}{100} = \frac{3}{5}$.

90. Three persons work independently on a problem. If the respective probabilities that they will solve it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$, then the probability that none can solve it

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{3}$ (D) None of these

Ans. : a

(a) Required probability $= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right)$

$= \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{2}{5}$.

91. A man and a woman appear in an interview for two vacancies in the same post. The probability of man's selection is $1/4$ and that of the woman's selection is $1/3$. What is the probability that none of them will be selected

(A) $\frac{1}{2}$ (B) $\frac{1}{12}$ (C) $\frac{1}{4}$ (D) None of these

Ans. : a

(a) Let E_1 be the event that man will be selected and E_2 the event that woman will be selected. Then

$P(E_1) = \frac{1}{4}$

so $P(\bar{E}_1) = 1 - \frac{1}{4} = \frac{3}{4}$ and $P(E_2) = \frac{1}{3}$

So $P(\bar{E}_2) = \frac{2}{3}$

Clearly E_1 and E_2 are independent events.

So, $P(\bar{E}_1 \cap \bar{E}_2) = P(\bar{E}_1) \times P(\bar{E}_2) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$.

92. Word '$UNIVERSITY$' is arranged randomly. Then the probability that both 'I' does not come together, is

(A) $\frac{3}{5}$ (B) $\frac{2}{5}$ (C) $\frac{4}{5}$ (D) $\frac{1}{5}$

Ans. : c

(c) Total number of ways $= \frac{10\,!}{2\,!}$

Favourable number of ways for $'I'$ come together is $9\,!$

Thus probability that $'I'$ come together

$= \frac{9\,! \times 2\,!}{10\,!} = \frac{2}{10} = \frac{1}{5}$.

Hence required probability $= 1 - \frac{1}{5} = \frac{4}{5}$.

93. If Mohan has 3 tickets of a lottery containing 3 prizes and 9 blanks, then his chance of winning prize are

(A) $\frac{34}{55}$ (B) $\frac{21}{55}$ (C) $\frac{17}{55}$ (D) None of these

Ans. : a

(a) Mohan can gets one prize, 2 prizes or 3 prizes and his chance of failure means he get no prize.

Number of total ways $= {}^{12}C_3 = 220$

Favourable number of ways to be failure $= {}^{9}C_3 = 84$

Hence required probability $= 1 - \frac{84}{220} = \frac{34}{55}$.

94. A box contains 25 tickets numbered $1, 2, \ldots\ldots. 25$. If two tickets are drawn at random then the probability that the product of their numbers is even, is

(A) $\frac{11}{50}$ (B) $\frac{13}{50}$ (C) $\frac{37}{50}$ (D) None of these

Ans. : c

(c) Required probability is $1 - P$ (Both odd numbers are chosen)

$= 1 - \frac{{}^{13}C_2}{{}^{25}C_2} = 1 - \frac{13 \,.\, 12}{25 \,.\, 24} = \frac{37}{50}$.

95. If four persons are chosen at random from a group of 3 men, 2 women and 4 children. Then the probability that exactly two of them are children, is

(A) $\frac{10}{21}$ (B) $\frac{8}{63}$ (C) $\frac{5}{21}$ (D) $\frac{9}{21}$

Ans. : a

(a) Total number of ways $= {}^9C_4$,

2 children are chosen in ${}^4C_2$ ways and other 2 persons are chosen in ${}^5C_2$ ways.

Hence required probability $= \frac{{}^4C_2 \times {}^5C_2}{{}^9C_4} = \frac{10}{21}$.

96. A bag contains 3 red, 7 white and 4 black balls. If three balls are drawn from the bag, then the probability that all of them are of the same colour is

(A) $\frac{6}{71}$ (B) $\frac{7}{81}$ (C) $\frac{10}{91}$ (D) None of these

Ans. : c

(c) Required probability $= \frac{{}^3C_3 + {}^7C_3 + {}^4C_3}{{}^{14}C_3}$

$= \frac{1+35+4}{14 \,.\, 13 \,.\, 2} = \frac{40}{14 \,.\, 26} = \frac{10}{91}$.

97. If out of 20 consecutive whole numbers two are chosen at random, then the probability that their sum is odd, is

(A) $\frac{5}{19}$ (B) $\frac{10}{19}$ (C) $\frac{9}{19}$ (D) None of these

Ans. : b

(b) The total number of ways in which 2 integers can be chosen from the given 20 integers ${}^{20}C_2$.

The sum of the selected numbers is odd if exactly one of them is given and one is odd.

$\therefore$ Favourable number of outcomes $= {}^{10}C_1 \times {}^{10}C_1$

$\therefore$ Required probability $= \frac{{}^{10}C_1 \times {}^{10}C_1}{{}^{20}C_2} = \frac{10}{19}$.

98. A man draws a card from a pack of 52 playing cards, replaces it and shuffles the pack. He continues this processes until he gets a card of spade. The probability that he will fail the first two times is

(A) $\frac{9}{16}$ (B) $\frac{1}{16}$ (C) $\frac{9}{64}$ (D) None of these

Ans. : c

(c) The required probability is given by

$$= \frac{^{39}C_1}{^{52}C_1} \times \frac{^{39}C_1}{^{52}C_1} \times \frac{^{13}C_1}{^{52}C_1}$$

$$= \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{9}{64}.$$

99. A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. Then P (neither A nor B) is equal to

(A) $2/3$ (B) $1/6$ (C) $5/6$ (D) $1/3$

Ans. : d

(d) $P($ neither A nor $B) = P(\bar{A} \cap \bar{B}) = P(\bar{A}) \, . \, P(\bar{B})$

$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

$P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore \; P(\bar{A}). P(\bar{B}) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}.$

100. The probabilities of a problem being solved by two students are $\frac{1}{2}, \frac{1}{3}$. Then the probability of the problem being solved is

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{1}{3}$ (D) 1

Ans. : a

(a) The probability that the problem is not being solved by any of two students

$= \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right) = \frac{1}{3}$ and

probability that the problem is solved $= 1 - \frac{1}{3} = \frac{2}{3}.$

101. 'X' speaks truth in 60% and 'Y' in 50% of the cases. The probability that they contradict each other narrating the same incident is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

Ans. : c

(c) Here $P(X) = \frac{3}{5}, P(Y) = \frac{1}{2}$

$\therefore$ Required probability $= P(X) \cdot P(\bar{Y}) + P(\bar{X})P(Y)$

$= \left(\frac{3}{5}\right)\left(1 - \frac{1}{2}\right) + \left(1 - \frac{2}{5}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{5} \cdot \frac{1}{2} + \frac{2}{5} \cdot \frac{1}{2} = \frac{1}{2}.$

102. The probability that a marksman will hit a target is given as $1/5$. Then his probability of at least one hit in 10 shots, is

(A) $1-\left(\frac{4}{5}\right)^{10}$ (B) $\frac{1}{5^{10}}$ (C) $1-\frac{1}{5^{10}}$ (D) None of these

Ans. : a

(a) It is obvious.

103. A bag contains 3 red and 7 black balls, two balls are taken out at random, without replacement. If the first ball taken out is red, then what is the probability that the second taken out ball is also red

(A) $\frac{1}{10}$ (B) $\frac{1}{15}$ (C) $\frac{3}{10}$ (D) $\frac{2}{21}$

Ans. : b

(b) We have total number of balls = 10

$\therefore$ Number of red balls = 3

and number of black balls = 7

and number of balls in the bag = $3+7=10$

$\therefore$ The probability for taking out one red ball out of 10 balls = $\frac{3}{10}$ and the probability for taking out one red ball out of remaining 9 balls = $\frac{2}{9}$

$\therefore$ Probability for both balls to be red

i.e., $p=\frac{3}{10}\times\frac{2}{9}=\frac{1}{15}$.

104. Two cards are drawn without replacement from a well-shuffled pack. Find the probability that one of them is an ace of heart

(A) $\frac{1}{25}$ (B) $\frac{1}{26}$ (C) $\frac{1}{52}$ (D) None of these

Ans. : b

(b) There are two conditions.

(i) When first is an ace of heart and second one is non-ace of heart

$=\frac{1}{52}\times\frac{51}{52}\Rightarrow\frac{1}{52}$

(ii) When first is non-ace of heart and second one is an ace of heart

$=\frac{51}{52}\times\frac{1}{51}=\frac{1}{52}$

$\therefore$ Required probability $=\frac{1}{52}+\frac{1}{52}=\frac{1}{26}$.

105. Find the probability that the two digit number formed by digits $1,2,3,4,5$ is divisible by 4 (while repetition of digit is allowed)

(A) $\frac{1}{30}$ (B) $\frac{1}{20}$ (C) $\frac{1}{40}$ (D) None of these

Ans. : d

(d) Total number of numbers $=(5)^2$

Favourable cases $=[12,24,32,44,52]$

$\therefore$ Required probability $=\frac{5}{25}=\frac{1}{5}$.

106. The probability that a leap year will have 53 Fridays or 53 Saturdays is

(A) $\frac{2}{7}$ (B) $\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{1}{7}$

Ans. : b

(b) There are 366 days in a leap year, in which 52 weeks and two days, The combination of 2 days -

Sunday -Monday, Monday -Tuesday, Tuesday -Wednesday, Wednesday -Thursday, Thursday -Friday, Friday -Saturday, Saturday -Sunday

P(53 Fridays) = $\frac{2}{7}$; P(53 Saturdays) $= \frac{2}{7}$

P(53 Fridays and 53 Saturdays) $= \frac{1}{7}$

$\therefore$ P(53 Fridays or Saturdays) = P(53 Fridays) + P(53 Saturdays) $-P$(53 Fridays and Saturdays)

$= \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$.

107. The corners of regular tetrahedrons are numbered $1, 2, 3, 4$. Three tetrahedrons are tossed. The probability that the sum of upward corners will be 5 is

(A) $\frac{5}{24}$ (B) $\frac{5}{64}$ (C) $\frac{3}{32}$ (D) $\frac{3}{16}$

Ans. : c

(c) Required combinations are $(2, 2, 1), (1, 2, 2), (2, 1, 2), (1, 3, 1,), (3, 1, 1)$ and $(1, 1, 3)$

$\therefore$ Required probability $= \frac{6}{4^3} = \frac{6}{64} = \frac{3}{32}$.

108. A person can kill a bird with probability $3/4$. He tries 5 times. What is the probability that he may not kill the bird

(A) $\frac{243}{1024}$ (B) $\frac{781}{1024}$ (C) $\frac{1}{1024}$ (D) $\frac{1023}{1024}$

Ans. : c

(c) Person has to miss all times probability of it will be $\left(\frac{1}{4}\right)^5 = \frac{1}{1024}$.

109. Two dice are thrown together. The probability that at least one will show its digit 6 is

(A) $\frac{11}{36}$ (B) $\frac{36}{11}$ (C) $\frac{5}{11}$ (D) $\frac{1}{6}$

Ans. : a

(a) Number of ways $= 6 \times 6 = 36$

Sample space = $\left\{\begin{matrix} (6,\ 1)\ (6,\ 2)\ (6,\ 3)\ (6,\ 4) \\ (6,\ 5)\ (1,\ 6)\ (2,\ 6)\ (3,\ 6) \\ (4,\ 6)\ (5,\ 6)\ (6,\ 6) \end{matrix}\right\}$

Probability of at least one 6

$= P(\text{one } 6) + P(\text{both } 6)$

$= \frac{10}{36} + \frac{1}{36} = \frac{11}{36}.$

110. Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to

(A) 4 (B) 3 (C) 1 (D) 2

Ans. : a

Total number of ways $= 6^3 = 216$

Favourable outcomes ${}^6p_3 = 120$

$\Rightarrow \text{Probability} = \frac{120}{216} = \frac{5}{9}$

$\Rightarrow p = 5, q = 9$

$\Rightarrow q - p = 4$

111. Let a die be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to:

(A) 30 (B) 90 (C) 15 (D) 60

Ans. : d

P (odd number 7 times) $= P$ (odd number 9 times)

$${}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$$

${}^nC_7 = {}^nC_9$

$\Rightarrow n = 16$

Required

$$P = {}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$$

$$= \frac{16 \cdot 15}{2} \times \frac{1}{2^{16}} = \frac{15}{2^{13}}$$

$$\Rightarrow \frac{60}{2^{15}} \Rightarrow k = 60$$

112. There are n letters and n addressed envelopes. The probability that all the letters are not kept in the right envelope, is

(A) $\frac{1}{n!}$ (B) $1 - \frac{1}{n!}$ (C) $1 - \frac{1}{n}$ (D) None of these

Ans. : b

(b) Required probability is $1 - P$ (All letters in right envelope) $= 1 - \frac{1}{n!}$

{As there are total number of n ! ways in which letters can take envelopes and just one way in which they have corresponding envelopes}.

113. Two dice are thrown 5 times, and each time the sum of the numbers obtained being 5 is considered a success. If the probability of having at least 4 successes is $\frac{\mathrm{k}}{3^{11}}$, then k is equal to

(A) 82 (B) 123 (C) 164 (D) 75

Ans. : b

Probability of success $=\frac{1}{9}=p$

Probability of failure $q=\frac{8}{9}$

$P(\text{ at least 4 success })=P(4 \text{ success })+P(5 \text{ success })$

$={}^5C_4 p^4 q+{}^5C_5 p^5=\frac{41}{3^{10}}=\frac{123}{3^{11}}$

$k=123$

114. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colours is q. If $p: q=m: n$, where m and n are coprime, then $m+n$ is equal to . . . .

(A) 15 (B) 14 (C) 13 (D) 12

Ans. : b

$p=\frac{{}^6C_1}{6 \times 6}=\frac{1}{6}$

$q=\frac{{}^6C_1 \times {}^5C_1 \times 4}{6 \times 6 \times 6 \times 6}=\frac{5}{54}$

$\therefore p: q=9: 5 \Rightarrow m+n=14$

115. If an unbiased die, marked with $-2,-1,0,1,2,3$ on its faces, is through five times, then the probability that the product of the outcomes is positive, is :

(A) $\frac{881}{2592}$ (B) $\frac{521}{2592}$ (C) $\frac{440}{2592}$ (D) $\frac{27}{288}$

Ans. : b

Either all outcomes are positive or any two are negative.

Now, $p=P(\text{positive})=\frac{3}{6}=\frac{1}{2}$

$q=p(\text{negative })=\frac{2}{6}=\frac{1}{3}$

Required probability

$={}^5C_5\left(\frac{1}{2}\right)^5+{}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^3+{}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{1}{2}\right)^1$

$=\frac{521}{2592}$

116. Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space $[0, 60]$ is less than or equal to a. If $P(A) = \frac{11}{36}$, then a is equal to ....

(A) 100 (B) 0.1 (C) 15 (D) 10

Ans. : d

$|x - y| < a \Rightarrow -a < x - y < a$

$\Rightarrow x - y < a$ and $x - y > -a$

$P(A) = \frac{\text{ar }(OACDEG)}{(OBDF)}$

$= \frac{\text{ar }(OBDF) - \text{ar }(ABC) - \text{ar }(EFG)}{\text{ar }(OBDF)}$

$\Rightarrow \frac{11}{36} = \frac{(60)^2 - \frac{1}{2}(60-a)^2 - \frac{1}{2}(60-a)^2}{3600}$

$\Rightarrow \quad 1100 = 3600 - (60 - a)^2$

$\Rightarrow \quad (60 - a)^2 = 2500 \Rightarrow 60 - a = 50$

$\Rightarrow \quad a = 10$

117. Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S = \{x \in Z : x(66 - x) \geq \frac{5}{9}M\}$ and the event $A = \{x \in S : x$ is a multiple of 3 $\}$. Then $P(A)$ is equal to

(A) $\frac{15}{44}$ (B) $\frac{1}{3}$ (C) $\frac{1}{5}$ (D) $\frac{7}{22}$

Ans. : b

$M = 33 \times 33$

$x(66 - x) \geq \frac{5}{9} \times 33 \times 33$

$11 \leq x \leq 55$

$A : \{12, 15, 18, ...54\}$

$P(A) = \frac{15}{45} = \frac{1}{3}$

118. Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting atmost two heads is

(A) $\frac{275}{6^5}$ (B) $\frac{36}{5^4}$ (C) $\frac{181}{5^5}$ (D) $\frac{46}{6^4}$

Ans. : d

$P(H) = x, P(T) = 1 - x$

$P(4H, 1T) = P(5H)$

${}^5C_1(x)^4(1 - x)^1 = {}^5C_5x^5$

$5(1 - x) = x$

$6x = 5 = 0 \quad x = \frac{5}{6}$

$P(\text{ atmost } 2H)$

$= P(OH, 5T) + P(1H, 4T) + P(2H, 3T)$

$= {}^5C_0\left(\frac{1}{6}\right)^5 + {}^5C_1\frac{5}{6}\cdot\left(\frac{1}{6}\right)^4 + {}^5C_2\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^3$

$= \frac{1}{6^5}(1 + 25 + 250) = \frac{276}{6^5}$

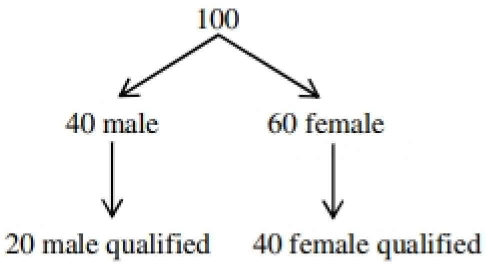
$= \frac{46}{6^4}$

119. Out of 60% female and 40% male candidates appearing in an exam, 60% candidates qualify it. The number of females qualifying the exam is twice the number of males qualifying it. A candidate is randomly chosen from the qualified candidates. The probability, that the chosen candidate is a female, is.

(A) $\frac{13}{16}$ (B) $\frac{11}{16}$ (C) $\frac{23}{32}$ (D) $\frac{2}{3}$

Ans. : d

Probability that chosen candidate is female $= \frac{40}{60} = \frac{2}{3}$



120. Let $S = \{1, 2, 3, \ldots, 2022\}$. Then the probability, that a randomly chosen number n from the set S such that $\mathrm{HCF}(n, 2022) = 1$, is.

(A) $\frac{128}{1011}$ (B) $\frac{166}{1011}$ (C) $\frac{127}{337}$ (D) $\frac{112}{337}$

Ans. : d

Total number of elements $= 2022$

$2022 = 2 \times 3 \times 337$

$\mathrm{HCF}(n, 2022) = 1$

is feasible when the value of ' n ' and 2022 has no common factor.

$A =$ Number which are divisible by 2 from

$\{1, 2, 3\ldots.2022\}$

$n(A) = 1011$

$B =$ Number which are divisible by 3 by 3

from $\{1, 2, 3\ldots.2022\}$

$n(B) = 674$

$A \cap B =$ Number which are divisible by 6

from $\{1, 2, 3\ldots\ldots.2022\}$

$6, 12, 18\ldots\ldots\ldots, 2022$

$337 = n(A \cap B)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 1011 + 674 - 337$

$= 1348$

$C =$ Number which divisible by 337 from

$\{1, \ldots\ldots .1022\}$

Total elements which are divisible by 2 or 3 or 337 $= 1348 + 2 = 1350$

Favourable cases $=$ Element which are neither divisible by 2, 3 or 337

$= 2022 - 1350$

$= 672$

Required probability $= \frac{672}{2022} = \frac{112}{337}$

*** Given section consists of questions of 3 marks each.** **[60]**

121. box contains 6 red marbles numbered 1 through 6 and 4 white marble numbered form 12 through 15. find the probability that a marble drawn is:

i. White

ii. White and odd numbered

iii. Even numbered

iv. Red or even numbered.

Ans. : We have 6 red marble numbered 1 - 6 and we have 4 white marble numbered 12 - 15 one marble is tobe drawn

$\therefore \mathrm{n(s)} = {}^{10}\mathrm{C}_1$

i. E be event of getting white marble,

$\therefore \mathrm{n(E)} = {}^{4}\mathrm{C}_1$

$\therefore \mathrm{P(E)} = \frac{{}^{4}\mathrm{C}_1}{{}^{10}\mathrm{C}_1} = \frac{4}{10} = \frac{1}{5}$

ii. E be the event of getting white marble with odd numbered marble.

$\therefore \mathrm{E} = \{13,\ 15\}$

$\Rightarrow \mathrm{n(E)} = 2$

$\mathrm{P(E)} = \frac{2}{10} = \frac{1}{5}$

iii. E be the event of getting even numbered marble.

$\therefore \mathrm{E} = \{2,\ 4,\ 6,\ 12,\ 24\}$

$\Rightarrow \mathrm{n(E)} = 5$

$\mathrm{P(E)} = \frac{5}{10} = \frac{1}{2}$

iv. E_1 be the event of getting red marble.

$\therefore \mathrm{P(E)}_1 = \frac{5}{10}$ [as in (ii)]

$\therefore (\mathrm{E}_1 \cap \mathrm{E}_2) = \text{even numbered marble} = \{2,\ 4,\ 6\}$

$\Rightarrow \mathrm{n}(\mathrm{E}_1 \cap \mathrm{E}_2) = 3$

$\Rightarrow \mathrm{P}(\mathrm{E}_1 \cap \mathrm{E}_2) = \frac{3}{10}$

$\therefore$ by law of addition,

$\Rightarrow P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{6}{10} + \frac{5}{10} - \frac{3}{10} = \frac{8}{10}$

$= \frac{4}{5}$

122. A box contains 10 white, 6 red and 10 black balls. A ball is drawn at random from the box. What is the probability that the ball drawn is either white or red?

Ans. : Let W be the event of drawing white ball

$\therefore p(W) = \frac{10}{26}$

Let R be the event of drawing red ball

$\therefore p(R) = \frac{6}{26}$

$\therefore P(W \cup R) = p(W) + p(R) - P(W \cap R)$ [$\because$ W and R are mutually exclusive case]

$\therefore P(W \cap R) = 0$

$= \frac{10}{26} + \frac{6}{26} - 0$

$= \frac{16}{26}$

$= \frac{8}{13}$

123. The probability that a person will travel by plane is $\frac{3}{5}$ and that he will travel by trains is $\frac{1}{4}$. What is the probability that he (she) will travel by plane or train?

Ans. : Let A be the event that the person travel by plane

$P(A) = \frac{3}{5}$

Let B be the event that the person travel by train

$P(B) = \frac{1}{4}$

$\therefore$ A and B are mutually exclusive case.

$\therefore P(A \cup B) = P(A) + P(B)$

$= \frac{3}{5} + \frac{1}{4}$

$= \frac{17}{20}$

124. In a race, the odds in favour of horses A, B, C, D are 1 : 3, 1 : 4, 1 : 5 and 1 : 6 respectively. Find probability that one of them wins the race.

Ans. : We have,

$P(A) : P(\bar{A}) = 1 : 3$

$\Rightarrow P(A) = \frac{1}{4}$

$P(B) : P(\bar{B}) = 1 : 4$

$\Rightarrow P(B) = \frac{1}{5}$

$P(C) : P(CB) = 1 : 5$

$\Rightarrow P(C) = \frac{1}{6}$

$P(D) : P(D) = 1 : 6$

$\Rightarrow P(D) = \frac{1}{7}$

$\therefore$ Probability that atleast one of them wins is given by $P(A \cup B \cup C \cup D)$

$= P(A) + P(B) + P(C) + P(D)$

$= \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$

$= \frac{319}{420}$

125. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Ans. : Let A be the event that choosing student who passed the first exam.

$\therefore p(A) = 0.8$

Let B be the event that choosing student who passed the 2nd exam.

$p(B) = 0.7$

$n(A \cup B) =$ number of students who passed atleast one of the two exams

$\Rightarrow p(A \cup B) = 0.95$

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$p(A \cap B) = p(A) + p(B) - p(A \cup B)$

$= 0.8 + 0.7 - 0.95$

$= -0.95 + 1.5$

$= 1.5 - 0.95$

$= 0.55$

126. There are four men and six women on the city councils. if one council member is selected for a committee at random, how likel6y is that it is a women?

Ans. : There are four men and six women on the city councils.

$\because$ one council member is selected for a committe.

$\therefore n(S) = {}^{10}C_1 = 10$

Let E be the events that is a women,

$\therefore n(E) = {}^{6}C_1 = 6$

$\therefore P(E) = \frac{6}{10} = \frac{3}{5}$

127. A box contains 30 bolts and 40 nuts. Half of the bolts and half of the nuts are rusted. If two items are drawn at random, what is the probability that either both are rusted or both are bolts?

Ans. : Box 30-bolts
40-nuts
Half the bolts and nuts are rusted
∴ rusted bolts = 15
∴ rusted bolts = 20
Since two items are drawn
$\therefore n(S) = {}^{70}C_2$
Let A be the event of choosing rusting item
$\therefore p(A) = \frac{{}^{35}C_2}{{}^{70}C_2}$
$= \frac{35 \times 34}{70 \times 69}$
Let B be the event of choosing bolts
$\therefore p(B) = \frac{{}^{30}C_2}{{}^{70}C_2}$
$= \frac{30 \times 29}{70 \times 69}$
Also, $n(A \cap B) = 15$ [bolts that are rusted]
$\therefore p(A \cap B) = \frac{{}^{15}C_2}{{}^{70}C_2}$
$= \frac{15 \times 14}{70 \times 69}$
$\therefore p(A \cup B) = p(A) + p(B) - p(A \cap B)$
$= \frac{35 \times 34}{70 \times 69} + \frac{30 \times 29}{70 \times 69} - \frac{15 \times 14}{70 \times 69}$
$= \frac{1850}{4830}$
$= \frac{185}{483}$

128. In a lottary, a person chooses six different numbers at random from 1 to 20, and if these six numbers match with six numbers already fixed by the lottery commitee, he wins the prize what is a probability of winning the prize in the game?

Ans. : As six number has been choose from 1 to 20 numbers
$\therefore \ {}^{20}C_6$
Let E be the event that six number choose in matched with the given number
[As winning number is fixed]
$\Rightarrow n(E) = 1$
$\therefore P(E) = \frac{1}{{}^{20}C_6}$
$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{20 \times 19 \times 18 \times 17 \times 16 \times 15}$
$= \frac{1}{38760}$

129. Tickets numbered form 1 to 20 are mixed up together and then a ticket is drawn at random,what is the probability that the ticket has a number which is a multiple of 3 or 7?

Ans. : Since one ticket is drawn form a mixed numbers (1 to 20) tickets.

$\therefore n(S) = {}^{20}C_1 = 20$

Let E be the events of getting ticket which has numbers that is multiple of 3 or 7.

$\therefore E = \{3, 6, 7, 9, 12, 14, 15, 18\}$

$\therefore n(E) = 8$

$\therefore P(E) = \frac{8}{20} = \frac{2}{5}$

$\therefore P(E) = \frac{2}{5}$

130. A bag contains tickets numbered from 1 to 20. Two tickets are drawn. Find the probability that (i) both the tickets have prime numbers on them (ii) on one there is a prime number and on the other there is a multiple of 4.

Ans. : Bag has tickets numbered from 1 to 20 two tickets are drawn

$\Rightarrow n(S) = {}^{20}C_2$

i. Let E be the event that both the tickets have prime number on them

$\therefore n(E) = {}^{8}C_2 = 56$ [as there are 8 prime numbers between 1 to 2 as 2, 3, 5, 7, 11, 13, 17, 19]

$\therefore p(E) = \frac{56}{{}^{20}C_2} = \frac{56}{20 \times 19} = \frac{14}{95}$

ii. Let E be the event that one tickets has prime numbers and other has multiple of 4.

$\therefore n(E) = 8 \times 5 = 40$

$p(E) = \frac{40}{{}^{20}C_2} = \frac{40 \times 2}{20 \times 19} = \frac{4}{19}$ [$\because$ (4, 8, 12, 16, 20) are multiples of 4]

131. Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have nonadjacent desks?
[**Hint:** First find the probability that the couple has adjacent desks, and then subtract it from 1]

Ans. : Number of desk occupied by one couple = 1

Only (4 + 1) = 5 persons to be assigned.

$\therefore$ Number of ways of assigning these 5 persons

5! × 2!

Total number of ways of assigning these 6 persons = 6!

$\therefore$ Probability that a couple has adjacent desk

$= \frac{5! \times 2!}{6!} = \frac{1}{3}$

So, the probaility that the married couple will have no-adjacent desks

$= 1 - \frac{1}{3} = \frac{2}{3}$

Hence, the required probability $= \frac{2}{3}$

132. An experiment consists of rolling a die until a 2 appears.
How many elements of the sample space correspond to the event that the 2 appears not later than the k^{th} roll of the die?
[**Hint:** 1 + 5 + 52 + ... + 5^{k-1}]

Ans. : Number of outcomes when die is thrown is ‘6’
we consider that 2 appears not later than K th roll of the die, then 2 comes before A^{th} roll.
If 2 appears in first roll, number of ways = 1 If 2 appears in second roll, number of ways = 5 × 1 (as first roll does not result in 2)
If 2 appears in third roll, number of ways = 5 × 5 × 1 (as first two rolls do not result in 2)
Similarly if 2 appears in $(k - l)^{th}$ roll, number of ways = [5 × 5 × 5 ...... (k - 1) times] × 1 = 5^{k-1} Possible outcomes if 2 appears before k^{th} roll = $1 + 5 + 5^2 + 5^3 + \ldots\ldots + 5^{k-l}$

$= \frac{1(5^k - 1)}{5 - 1} = \frac{5^k - 1}{4}$

133. If the letters of the word ASSASSINATION are arranged at random. Find the Probability that:
Two I’s and two N’s come together.

Ans. : We have word 'ASSASSINATION'.
Numberof letter = 13
Letters are 3A's, 4S's, 2I's 1T's and 1O's
Total number of ways these letters can be arranged $= n(S) = \frac{13!}{3!4!2!2!}$
If 2 I's and 2N's come together, then these as 10 alphabets.
I.e., (IINN), A, A, A, S, S, S, S, T, O
Number of words when 2 I's and 2 N's are come together

$= \frac{10!}{3!4!} \times \frac{4!}{2!2!}$

$\therefore$ Required probability $= \frac{\frac{10!4!}{3!4!2!2!}}{\frac{13!}{3!4!2!2!}}$

$= \frac{4!}{13 \times 12 \times 11} = \frac{2}{143}$

134. If the letters of the word ASSASSINATION are arranged at random. Find the Probability that:
All A’s are not coming together.

Ans. : We have word 'ASSASSINATION'.
Numberof letter = 13

Letters are 3A's, 4S's, 2I's 1T's and 1O's

Total number of ways these letters can be arranged $= n(S) = \frac{13!}{3!4!2!2!}$

If all A's are coming together, then are 11 alphabets

I.e., (AAA), S, S, S, S, I, I, N, N, T, O

$\therefore$ Number of word when all A's come together $= \frac{11!}{4!2!2!}$

$\therefore$ Probability when all A's come together

$$= \frac{\frac{11!}{4!2!2!}}{\frac{13!}{4!3!2!2!}} = \frac{3!}{13 \times 12} = \frac{1}{26}$$

Then the probability thatb all A's does not come together

$$= 1 - \frac{1}{26} = \frac{25}{26}$$

135. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the Probability that:

a. All the three balls are white.

b. All the three balls are red.

c. One ball is red and two balls are white.

Ans. : Three balls are draw at random

$\therefore$ $n(S) = {}^{13}C_3$

a. P(All the three balls are white)

$$= \frac{{}^5C_{13}}{{}^{13}C_3} = \frac{\frac{5!}{3!2!}}{\frac{13!}{3!10!}} = \frac{10}{\frac{13 \times 12 \times 11}{6}} = \frac{5}{143}$$

b. P(All the three balls are red)

$$= \frac{{}^8C_3}{{}^{13}C_3} = \frac{\frac{8 \times 7 \times 6}{6}}{\frac{13 \times 12 \times 11}{6}} = \frac{8 \times 7 \times 6}{13 \times 12 \times 11} = \frac{28}{143}$$

c. P(One ball is red and two balls are white)

$$= \frac{{}^8C_1 \times {}^5C_2}{{}^{13}C_3} = \frac{8 \times 10}{\frac{13 \times 12 \times 11}{6}} = \frac{40}{143}$$

136. One urn contains two black balls (labelled B_1 and B_2) and one white ball. A second urn contains one black ball and two white balls (labelled W_1 and W_2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball:

a. Write the sample space showing all possible outcomes.

b. What is the probability that two black balls are chosen?

c. What is the probability that two balls of opposite colour are chosen?

Ans. : It is given that one of the two urn is chosen, then a ball is randomly chosen from the urn, then a second ball is chosen at random from the same urn without replacing the first ball.

a. Sample space S = {B_1B_2, B_2B_1, B_1W, WB_1, B_2W, WB_2,BW_1, W_1B, BW_2, W_2B, W_1W_2, W_2W_1}

$\therefore\ n(S) = 12$

b. If two black ball are chosen, then favourable cases are B1B2 and B2B1.

$\therefore$ Reduired probability $= \frac{2}{12} = \frac{1}{6}$

c. If two balls of opposite colours are chosen, the favourable cases are B_1W, WB_1, B_2W, WB_2, BW_1, W_1B, BW_2 and W_2B.

$\therefore$ Reduired probability $= \frac{8}{12} = \frac{2}{3}$

137. If the letters of the word ALGORITHM are arranged at random in a row what is the probability the letters GOR must remain together as a unit?

Ans. : Word ALGORITHM has 9 letters.

If GOR remain together, then it will remain together.

$\therefore$ Number of letter AL**GOR**ITHM = 6 + 1 = 7

Number of words = 7!

and the total number of word from ALGORITHM = 9!

So, th required probability $= \frac{7!}{9!} = \frac{7!}{9 \cdot 8 \cdot 7!} = \frac{1}{72}$

Hence, the required probability $= \frac{1}{72}$

138. If the letters of the word ASSASSINATION are arranged at random. Find the Probability that:

Four S's come consecutively in the word.

Ans. : We have word 'ASSASSINATION'.

Numberof letter = 13

Letters are 3A's, 4S's, 2I's 1T's and 1O's

Total number of ways these letters can be arranged $= n(S) = \frac{13!}{3!4!2!2!}$

If for S's come consecutively in the word then we consider these 4S's as 1 group.

So, now number of letters is 10 i.e., (SSSS), A, A, A, I, I, N, N, T,O

$\therefore\ n(E) = \frac{10!}{3!2!2!}$

$\therefore$ Required probability $= \frac{\frac{10!}{3!2!2!}}{\frac{13!}{3!4!2!2!}}$

$= \frac{4!}{13 \times 12 \times 11} = \frac{2}{143}$

139. If the letters of the word ASSASSINATION are arranged at random. Find the Probability that:

No two A's are coming together.

Ans. : We have word 'ASSASSINATION'.

Numberof letter = 13

Letters are 3A's, 4S's, 2I's 1T's and 1O's

Total number of ways these letters can be arranged $= n(S) = \frac{13!}{3!4!2!2!}$

If no two A's are together, then first we arrange the alphabets other than A's

I.e., S, S, S, S, I, I, N, N, T, O

These letters can be arrangerd in $\frac{10!}{4!2!2!}$ ways.

xS xS xS xS xI xI xN xN xT xOx

Arrangement of these letters creates eleven gaps shown as 'x'.

Three gaps for three A's can be sslected in $^{11}C_3$ ways.

$\therefore$ Total number of words when no two A's together

$= {}^{11}C_3 \times \frac{10!}{4!2!2!} = \frac{11!}{3!8!} \times \frac{10!}{4!2!2!}$

$\therefore$ The probability that no two A's come together

$= \frac{\frac{11! \times 10!}{3!8!4!2!2!}}{\frac{13!}{4!3!2!2!}} = \frac{10!}{8! \times 13 \times 12} = \frac{10 \times 9}{13 \times 12} = \frac{15}{26}$

140. Suppose an integer from 1 through 1000 is chosen at random, find the probability that the integer is a multiple of 2 or a multiple of 9.

Ans. : We have integers 1, 2, 3, ..., 1000

We have integers 1, 2, 3, ..., 1000

n(S) = 1000

Number of integers which are multiple of 2 = 500 Let the number of integers which are multiple of 9 be n.

n^{th} term = 999

⇒ 9 + (n -1)9 = 999

⇒ 9 + 9n - 9 = 999

⇒ n = 111

From 1 to 1000, the number of multiples of 9 is 111.

The multiple of 2 and 9 both are 18, 36, ..., 990.

Let m be the number of terms in above series.

m^{th} term = 990

⇒ 18 + (m - 1)18 = 990

⇒ 18 + 18m - 18 = 990

⇒ m = 55

Number of multiples of 2 or 9 = 500 + 111 - 55 = 556 = n(E)

$\therefore$ Required probability $= \frac{n(E)}{n(S)}$

$= \frac{556}{1000} = 0.556$

*** Given section consists of questions of 5 marks each.** **[50]**

141. A class consists of 10 boys and 8 girls. Three students are selected at random. What is the probability that the selected group has

i. All boys?
ii. All girls?
iii. 1 boys and 2 girls?
iv. At least one girl?
v. At most one girl?

Ans. : 10 boys

8 girls

Three student are selected at random

$n(S) = {}^{18}C_3$

i. E be the event that the group has all boys

$\therefore n(E) = {}^{18}C_3$

$\therefore p(E) = \frac{{}^{10}C_3}{{}^{18}C_3}$

$= \frac{10 \times 9 \times 8}{18 \times 17 \times 16}$

$= \frac{5}{34}$

ii. E be the event that the group has all girls

$\therefore n(E) = {}^{8}C_3$

$\therefore p(E) = \frac{{}^{8}C_3}{{}^{18}C_3}$

$= \frac{8 \times 7 \times 6}{18 \times 17 \times 16}$

$= \frac{7}{102}$

iii. E be the event that the group has one boy and two girls

$\therefore n(E) = {}^{8}C_3 \times {}^{10}C_2$

$\therefore p(E) = \frac{{}^{8}C_1 \times {}^{10}C_2}{{}^{18}C_3}$

$= \frac{35}{102}$

iv. E be the event that atleast one girls in the group

$E = (1,\ 2,\ 3)\text{girls}$

$\therefore n(E) = {}^{8}C_1 \times {}^{10}C_2 + {}^{8}C_2 \times {}^{10}C_1 + {}^{8}C_3 \times {}^{10}C_0$

$p(E) = \frac{{}^{8}C_1 \times {}^{10}C_2 + {}^{8}C_2 \times {}^{10}C_1 + {}^{8}C_3}{{}^{18}C_3}$

$= \frac{29}{34}$

v. E be the event that almost one girls in the group

$E = (0,\ 1,)\text{girls}$

$\therefore n(E) = {}^{8}C_0 \times {}^{10}C_3 + {}^{8}C_1 \times {}^{10}C_2$

$$p(E) = \frac{{}^{10}C_3 + 8 \times {}^{10}C_2}{{}^{18}C_3}$$

$$= \frac{10}{17}$$

142. A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn at random. from the box, what is the probability that:

i. All are blue?

ii. At least one is green?

Ans. : 10 Red, 20 Blue, 30 Green

i. All 5 are blue

$$= \frac{{}^{20}C_5 \times {}^{40}C_0}{{}^{60}C_5} = \frac{34}{11977}$$

ii. Atleast one green = 1 - no green

Different combination one possible for no green case are 5B, 1R, 2R, 3B, 3R, 2B, 4R, 2B, 5R,

$5B = {}^{20}C_5$

$1R\ 4B = {}^{10}C_1 \times {}^{20}C_4$

$2R\ 3B = {}^{10}C_2 \times {}^{20}C_3$

$3R\ 2B = {}^{10}C_3 \times {}^{20}C_2$

$4R\ 1B = {}^{10}C_1 \times {}^{20}C_1$

$5R = {}^{10}C_5$

Atleast one green = 1 - no green

$$= 1 - \frac{{}^{20}C_5 + {}^{10}C_1 \times {}^{20}C_4 + {}^{10}C_2 \times {}^{20}C_3 + {}^{10}C_3 \times {}^{20}C_2 + {}^{10}C_4 \times {}^{20}C_1 + {}^{10}C_5}{{}^{60}C_5}$$

$$= \frac{4367}{4484}$$

143. 20 cards are numbered form 1 to 20. card is drawn at random. what is the probability that trhe number on the card is:

i. A multiple of 4?

ii. Not a multiple of 4?

iii. odd?

iv. Greather than 12?

v. Divisible by 5?

vi. Not a multiple of 6?

Ans. : We have 20 cards numbered from 1 to 20, one card is drawn at random

$\therefore n(S) = {}^{20}C_1 = 20$

i. Let E be the event that the number on the drawn cards is multiple of 4

$\therefore E = \{4, 8, 12, 16, 20\}$

$\therefore n(E) = 5$

$\therefore p(E) = \frac{5}{20} = \frac{1}{4}$

ii. Let E be the event that the number on the drawn cards is not multiple of 4

$\therefore \tilde{E}$ be the event that the number on the drawn cards is not multiple of 4

$\therefore \tilde{E} = \{4, 8, 12, 16, 20\}$

$\Rightarrow n(\tilde{E}) = 5$

$\therefore P(\tilde{E}) = \frac{5}{20} = \frac{1}{4}$

$\therefore P(E) = 1 - P(\tilde{E})$

$= 1 - \frac{1}{4} = \frac{3}{4}$

iii. Let E be the event that the number on the drawn cards is odd.

$\therefore E = \{1, 3, 5, 7, 13, 15, 17, 19\}$

$\therefore n(E) = 10$

$\Rightarrow P(E) = \frac{10}{20} = \frac{1}{2}$

iv. Let E be the event that the number on the drawn cards is greater than 12.

$\therefore E = \{13, 14, 15, 16, 17, 18, 19, 20\}$

$\therefore n(E) = 8$

$\Rightarrow P(E) = \frac{8}{20} = \frac{2}{5}$

v. Let E be the event that the number on the drawn cards is divisible by 5.

$\therefore E = \{5, 10, 15, 20\}$

$n(E) = 4$

$\therefore P(E) = \frac{4}{20} = \frac{1}{5}$

vi. Let E be the event that the number on the drawn cards is divisible by 6.

$\therefore \tilde{E}$ be the event that the number on the drawn cards is not divisible of 6

$\Rightarrow n(\tilde{E}) = 3$

$\therefore P(\tilde{E}) = \frac{3}{20}$

$P(E) = 1 - P(\tilde{E})$

$= 1 - \frac{3}{20} = \frac{17}{20}$

144. An integer is chosen at random from first 200 positive integers. Find the probability that the integer is divisible by 6 or 8.

Ans. : Let A be the event of choosing a positive integer divisible by 6

$\therefore A = \{6, 12, \ldots, 198\}$

$\Rightarrow n(A) = 33$

$\therefore p(A) = \frac{33}{200}$

Let B be the event of choosing a positive integer divisible by 8

$\therefore B = \{8,\ 16,\ ....,\ 200\}$
$\Rightarrow n(B) = 25$
$\therefore p(B) = \frac{25}{200}$
Also, $A \cap B = \{24,\ 28,\ ...,\ 192\}$
$\Rightarrow n(A \cap B) = 8$
$\therefore p(A \cap B) = \frac{8}{200}$
$\therefore p(A \cup B) = \frac{1}{4}$

145. Suppose an integer from 1 through 1000 is chosen at random, find the probability that the integer is a multiple of 2 or a multiple of 9.

Ans. : Number of multiples of 2 in 1 to 1000 are 500
Number of multiples of 9 in 1 to 1000 are 111
Out of 111, 55 are even numbers. So total favorables number are
500 + 56 = 556
Probability that integer is a multiple of 2 or a multiple of 9
$= \frac{556}{1000} = 0.556$

146. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English Examination is 0.75. What is the probability of passing the Hindi Examination?

Ans. : Let E be the event that student passed in english examination
$\therefore P(E) = 0.75$
Let H be the event that student passed in hindi examination
$\therefore P(H) = ?$
Also, $P(E \cap H) = 0.5$ and $P(\bar{E} \cap \bar{H}) = 0.1$
$\because P(\bar{E} \cap \bar{H}) = 1 - P(E \cup H)$
$\Rightarrow P(E \cup H) = 1 - 0.1$
$= 0.9$
Now,
$P(E \cup H) = P(E) + P(H) - P(E \cap H)$
$0.9 = 0.75 + P(H) - 0.5$
$P(H) = 0.90 - 0.25$
$= 0.65$

147. Five cards are drawn from form a pack of 52 cards. what is the chance that these 5 will contain:
i. Just one ace
ii. At least one ace.

Ans. : Since five card are drawn from a pack to 52 card

$= {}^{52}C_5$

i. Let E be the event that those five card contain exactly one ace.

$\therefore n(E) = {}^{4}C_1 \times {}^{48}C_4$

$\therefore n(E) = \dfrac{{}^{4}C_1 \times {}^{48}C_4}{{}^{52}C_5}$

$= \dfrac{4 \times 48 \times 47 \times 46 \times 45}{\frac{52 \times 51 \times 50 \times 49 \times 48}{5}}$

$= \dfrac{3243}{10829}$

ii. Let E be the event that five cards contain atleast one ace.

$\therefore E = \{1 \text{ or } 2 \text{ or } 3 \text{ or } 4\}$

$n(E) = \dfrac{{}^{4}C_1 \times {}^{48}C_4 + {}^{4}C_2 \times {}^{48}C_3 + {}^{4}C_3 \times {}^{48}C_2 + {}^{4}C_4 \times {}^{48}C_1}{{}^{52}C_5}$

$= \dfrac{4 \times \frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2 \times 1} + \frac{4 \times 3}{2} \times \frac{48 \times 47 \times 46}{3 \times 2 \times 1} + 4 \times \frac{48 \times 47}{2} + 48}{\frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1}}$

$= \dfrac{18472}{54145}$

148. In a large metropolitan area, the probabilities are 0.87, 0.36, 0.30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set, or both kinds of sets. What is the probability that a family owns either any one or both kinds of sets?

Ans. : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.87 + 0.36 - 0.30 = 0.93$

149. Match the proposed probability under Column C_1 with the appropriate written description under column C_2:

	C_1		**C_2**
	Probability		**Written Description.**
a.	0.95	i.	An incorrect assignment.
b.	0.02	ii.	No chance of happening.
c.	-0.3	iii.	As much chance of happening as not.
d.	0.5	iv.	Very likely to happen.
e.	0	v.	Very little chance of happening.

Ans. :

	C_1		**C_2**
	Probability		**Written Description.**
a.	0.95	iv.	Very likely to happen.
b.	0.02	v.	Very little chance of happening.

c.	-0.3	i.	An incorrect assignment.
d.	0.5	iii.	As much chance of happening as not.
e.	0	ii.	No chance of happening.

Solution:

1. 0.95 = Very likely to happen, so it is close to 1.
2. 0.02 = Very little chance of happening as the probability is very low.
3. -0.3 = an incorrect assignment because probability is never negative.
4. 0.5 = As much chance of happening as not because sum of chances of happening and not happening is one.
5. 0 = No chance of happening.

150. Four candidates A, B, C, D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B, and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, what are the probabilities that:
   a. C will be selected?
   b. A will not be selected?

Ans. : Given that A is twice as likely to be selected as B

I.e., $P(A) = 2P(B)$

and C is twice as likely to be selected as D

$\therefore\ P(C) = 2P(D)$

$\Rightarrow P(B) = 2P(D)$

$\Rightarrow \frac{P(A)}{2} = 2P(D)$

$\Rightarrow P(D)\frac{1}{4}P(A)$

Now B and C are given about the same chance

$\therefore\ P(B) = P(C)$

Since, sum of all probabilities = 1

$\therefore\ P(A) + P(B) + P(C) + P(D) = 1$

$\Rightarrow P(A) + \frac{P(A)}{2} + \frac{P(A)}{2} + \frac{P(A)}{4} = 1$

$\Rightarrow 4P(A) + 2P(A) + 2P(A) + P(A) = 4$

$\Rightarrow 9P(A) = 4$

$\Rightarrow P(A) = \frac{4}{9}$

a. P(C will be selected)

$= P(C) = P(B) = \frac{P(A)}{2}$

$= \frac{4}{9} \times \frac{1}{2} = \frac{2}{9}$

b. P(A will not be selected)

$P(A^{'}) = 1 - P(A)$

$= 1 - \frac{4}{9} = \frac{5}{9}$

----- Every failure is a new lesson, and every lesson is a key to new success.” -----